Cloud Publications
International Journal of Advanced Packaging Technology
2016, Volume 4, Issue 1, pp. 177-184
ISSN 2349-6665, Article ID Tech-576

Research Article
Open Access

Synchrosqueezing Transform for High Resolution Time-Frequency Analysis of Shock and Vibration Measurements

Divya Choudhary

Healthcare Packaging Consortium, Christian Brothers University, 650 East Parkway South, Memphis, TN, USA

Correspondence should be addressed to Divya Choudhary, dchodhry@cbu.edu

Publication Date: 26 February 2016

Article Link: http://technical.cloud-journals.com/index.php/IJAPT/article/view/Tech-576

Abstract This article presents the use of the synchrosqueezing transform (SST) for the time-frequency analysis of shock and vibration measurements. The time-frequency resolution properties of SST is demonstrated by analyzing acceleration measurements from pallets dropped from heights and vibration of pallets acquired in controlled laboratory settings. The article presents results that indicate that SST, in conjunction with continuous wavelet transform (CWT) based preprocessing for noise reduction, can be used as an effective spectral analysis tool for shock and vibration measurements.

Keywords *Synchrosqueezing Transform; Continuous Wavelet Transform; Shock; Vibration*

1. Introduction

Shock is a transient event defined as a mechanical disturbance characterized by a rise and decay of acceleration in a short period of time, while vibrations are random oscillations about a reference point, usually for a longer period of time [1]. Packaged products often undergo shock and vibration during distribution. An accurate simulation of the shock and vibration phenomenon enables effective testing of packaging components and provides direction for further improvement of packaging and transportation design. Hence, the understanding of spectral (frequency) components and time periods when the spectral components occur as a result of stimulus caused by shock and vibration is important. Given the definitions and observations made through measurements, shock and vibration are considered non-stationary processes. Non-stationary signals contain different frequency components at different periods of time. Wavelet Transform maps a temporal signal on to a 3-D time-frequency space and is used extensively to analyze non stationary signals [2, 3]. The time-frequency localization properties of wavelet basis functions and the mechanism of the transform process makes continuous wavelet transform (CWT) provide better interpretation of Shock and Vibration signals than techniques based on Fourier Transform, Short Time Fourier Transform and Shock Response Spectrum (SRS) [4]. Fourier Transform based spectral analysis only provides information about the frequency content of the signal but not the time periods when those frequencies are present. It is a 2-D representation of the signal, with power spectral density of the y axis and frequency on the x axis. The Shock Response Spectrum (SRS) is another approach to analyze shock data that assumes a model containing a set of single degree-of-freedom, mass-damper-spring oscillator subsystems that are excited by base motion [5]. Although STFT provides information in the time-frequency domain in a 3D format with coefficients/Power on the z-axis, time on the x-axis and frequency on the y-axis, the trade-off between time resolution and frequency resolution is non-optimum.

Synchrosqueezing transform (SST) is a method to further sharpen the time-frequency representation of the continuous wavelet transform (CWT). SST process starts with the implementation of the wavelet transform, followed by reallocation of the wavelet coefficients to improve the time-frequency representation [6]. Since wavelet transform is the first step to the computation of SST, as an intermediate step, between computation of wavelet coefficients and coefficient reallocation, wavelet based noise suppression techniques can be introduced to increase accuracy of the analysis. In this article, the use of SST with wavelet based noise suppression technique for time-frequency analysis of shock and vibration of pallets is demonstrated.

The subsequent sections in this article are as follows: In section 2, data collection methods and signal processing algorithms are described. Section 3 discusses the results of the analysis of the data and section 4 presents the conclusions drawn from this research.

2. Materials and Methods

In this section, first, a description of the shock and vibration experiment is provided. Next, the CWT and SST based signal processing for analyzing the data are presented. Finally, the software tools to implement the analysis are discussed.

2.1. Data Collection Procedure

The shock and vibration data collection was done in a controlled laboratory environment [4]. For recording shock data, a Lansmont Saver 3M30 recorder was used to measure acceleration versus time at 1000 samples/sec along three directions. It was attached to a pallet, which was raised and dropped from a certain height. In this experiment, a wooden pallet was dropped from 2 inches, 4 inches, 6 inches, 8 inches and 10 inches. Figure 1 shows the setup of the shock experiment. For each height, acceleration versus time was measured through the three channels of the shock recorder. Channel 3 measured the acceleration along the direction of the drop, while the other two channels measuring acceleration along the other two orthogonal directions. For measuring vibrational data, a wooden pallet was mounted on a vibration platform as shown in Figure 2. The Lansmont recorder was used to measure the vibrational acceleration versus time signal sampled at 1000 samples/sec along three orthogonal directions (x, y and z axis). A truck vibration simulation in accordance with ASTM D 4169 Truck Level I was utilized.

Figure 1: *Experimental Setup for Collection of Shock Data*

Figure 2: *Experimental Setup for Collection of Vibration data*

2.2. Signal Processing/Modeling Techniques

In this sub-section, the theoretical background and software tools CWT, SST in combination with noise suppression using CWT. In the context of this article, the time varying, zero-mean function $x(t)$ represents the acceleration versus time signal associated with the shock or vibration data. Further, the signal $x(t)$ is normalized by subtracting its mean value from the signal.

2.2.1. Continuous Wavelet Transform

Wavelet Transform represents a signal $x(t)$ as a weighted sum of basis functions referred to as wavelets. The weights correspond to the wavelet coefficients. The Continuous Wavelet Transform (CWT) of a signal $x(t)$ is given by [7]: $W(a,b)=\int x(t)\frac{1}{\sqrt{a}}\phi^*\left(\frac{t-b}{a}\right)$, where b is the translation parameter and a is the scale parameter. The basis function $\phi(t)$ is referred to as a mother wavelet. $\phi^*(t)$ is the complex conjugate of $\phi(t)$. The translation parameter, b, shifts $\phi(t)$ in time and the scale parameter, *s*, controls the temporal width of $\phi(t)$. The scale parameter is inversely related to frequency. An example of a mother wavelet function is a Morlet function. The Morlet wavelet is a complex valued function given by: $\phi(t) = e^{-2\pi^2 \frac{t^2}{z_0^2}} \left(e^{j2\pi t} - e^{2\pi^2 t^2}\right)$. The envelope factor z_o controls the number of oscillations in the wavelet with a typical value of z_o = 5 [8]. The Morlet basis function is used in this article for the computation of CWT.

The CWT is the correlation or the inner product of the signal $x(t)$ with various shifted and stretched/shrunken versions of the mother wavelet $\phi(t)$. It is this ability to manipulate the width (stretching or shrinking) of the mother wavelet and shift it along the time axis that makes the CWT time-frequency analysis effective. The plot of CWT coefficients for the signal $x(t)$ is a 3D plot. The x-axis corresponds to the time shift, b. The y-axis represents frequency *f* or scale *a*. The amplitude of the CWT coefficients is represented by the z-axis.

2.2.2. Synchrosqueezing Transform

SST further sharpens the time-frequency representation of wavelet transform by constructing a more concentrated representation. This is done by combining all wavelet coefficients corresponding to same instantaneous frequencies into one SST coefficient [9]. This process is referred to as frequency reallocation.

The steps involved in computing the SST coefficients is as follows [6][10]:

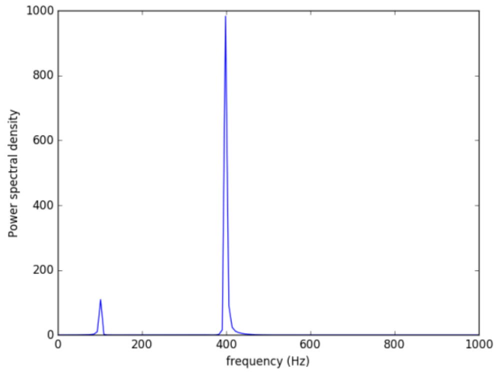
1) SST starts with the decomposition of the signal x(t) to determine the wavelet coefficients $W(a,b)$ as described in the section 2.2.1. The wavelet coefficients are however computed for discrete values, a_k and $a_k - a_{k-1} = (\Delta a)_k$ The instantaneous frequencies is computed, $\omega(a,b) = -i(W(a,b)^{-1}\frac{\partial}{\partial b}W(a,b)$. Note that $\omega = 2\pi f$ is the frequency expressed in radians/second.

2) Each point in the wavelet time-scale domain, located at co-ordinates (b, a), is mapped in to a corresponding point (b, ω) using the synchrosqueezing process. This process, synchrosqueezing transform $T_S(\omega, b)$, is computed at center ω_l of successive bins $\left[\omega_l - \frac{1}{2}\Delta\omega, \omega_l + \frac{1}{2}\Delta\omega\right]$, where $\omega_l - \omega_{l-1} = \Delta\omega$, by:
$$T_s(\omega_l, b) = (\Delta\omega)^{-1} \sum_{a_k:|\omega(a_k,b)-\omega_l|\leq\Delta\omega/2} W_s(a_k, b)a_k^{-3/2}(\Delta a)_k$$

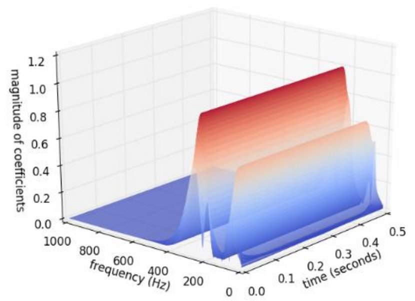
Measurement and environmental noise can be suppressed using wavelet transform based noise reduction techniques. Wavelets provide a sparse representation of the signal in that most of the energy in the signal is concentrated in small number of wavelet coefficients. Hence, noise can be reduced by setting to zero most of the small valued wavelet coefficients that fall below a certain hard threshold. In this research, the threshold is determined using the RiskShrink algorithm which mimics the performance of an oracle for selective wavelet reconstruction, minimizing the error between the estimated signal and the true signal [11]. This process of noise reduction is introduced as an intermediate step between step 1 and 2 in computation of SST described above.

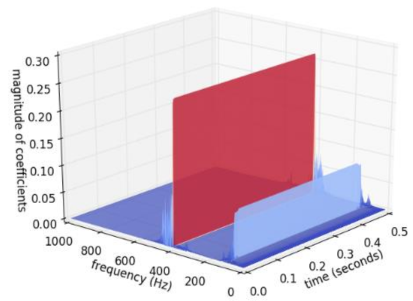
3. Results and Discussion

The shock and vibration data collected in the experiments are processed using SynchWave package implemented in R programming language CWT and SST with noise suppression [12]. Before the analysis of shock and vibration signal, to demonstrate the improvement in time-frequency resolution that SST provides over CWT, a well characterized signal $x(t) = sin(2\pi 100t) + 3\sin(2\pi 400t)$ sampled at 2000Hz is transformed by SST and CWT. Figure 3 shows the results of the transformations. The 3-D time-frequency representation is sharper with SST as indicated by the clearly resolved coefficients at focused at 100Hz and 400Hz. On the other hand, CWT spectrum is smeared over the region around 100Hz and 400Hz. The Fourier Transform based power spectral density (FT-PSD), Figure 3(a) based Power spectral density is shown to provide a clear picture of frequencies in the signal in a 2-D plot (PSD vs frequency).



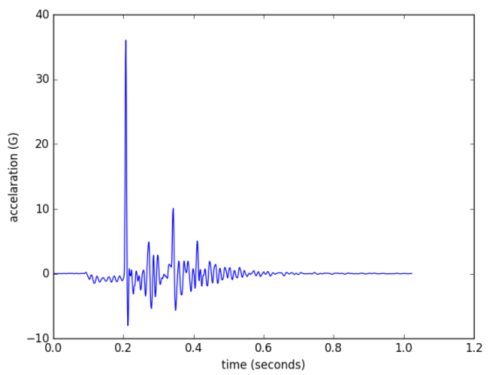
(a)

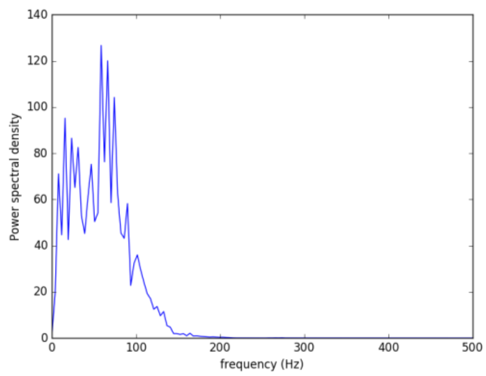




(b) (c)

Figure 3: *(a) Fourier Transform based PSD of sum of 100Hz and 400Hz sine wave (b (c) CWT*





(a) (b)

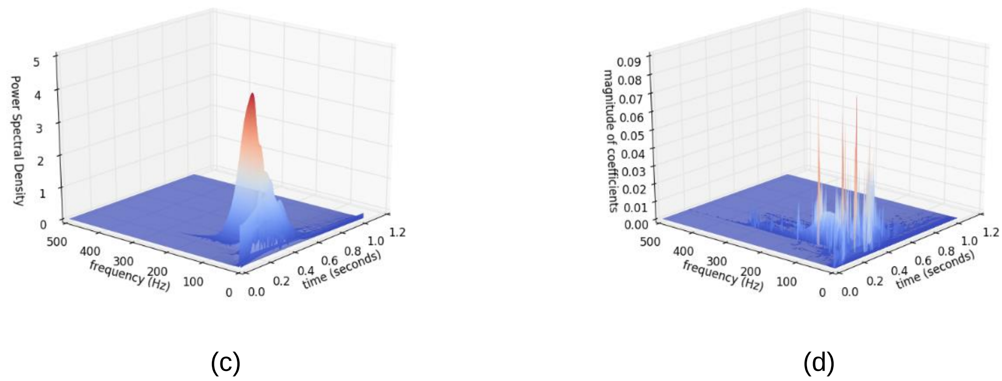


Figure 4: *(a) Acceleration Measurement of Shock Produced by 2 inch drop (b) Fourier Transform based PSD (c) CWT (d) SST*

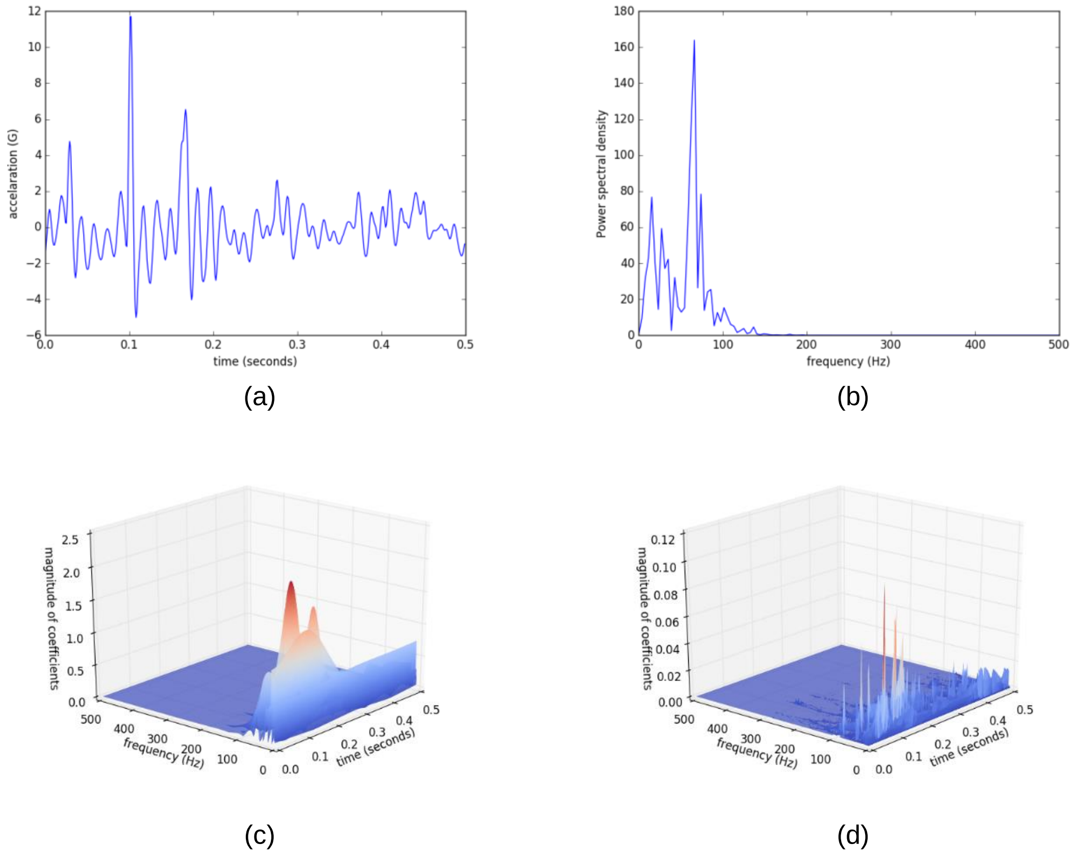


Figure 5: *(a) Acceleration Measurement of Shock Produced by 2 Inch Drop (b) Fourier Transform based PSD (c) CWT (d) SST*

Figure 4 shows the acceleration, FT-PSD, SST and CWT for the shock experiment for a 2 inch drop along the direction of the drop (z axis). Figure 5 shows the acceleration, FT-PSD, SST and CWT for the vibration measurement along the direction of the drop (z axis). Figures 4 and 5 are exemplar plots and the observations derived through these are consistent for other measurements from the experiment as well. For the shock signal, FT-PSD, CWT and SST shows peaks at 75 Hz. Several other frequency components are seen at between 10Hz and the peak at 75Hz. For FT-PSD, there is no information about the time periods when these frequencies are present. It is shown here to provide a clear view of the frequency components present in the signal. Both SST and CWT indicate that the dominant frequency component occurs approximately in a temporal region around 0.2 seconds. Uncertainty principles in time-frequency resolution dictates that the time instant when a specific frequency signal occurred can only be estimated up to certain accuracy. The CWT coefficients are smeared in over various frequency components, but the synchrosqueezing process described in section 2.2.2 allows SST to resolve those frequency components more clearly. Similarly, inferences can be made for vibrational data analysis as represented in Figure 5. The vibrational data analysis shows strong frequency components in the frequency band less than 100 Hz. CWT and SST show that these frequency components occur around 80Hz and other frequency components between 10 Hz and 50Hz. The resolution is once again better in the SST domain than the CWT with better resolved frequency components. The use of CWT noise suppression as an intermediate step in SST also reduces possible measurement or environmental noise in the time-frequency analysis.

4. Conclusion

This article introduces SST in conjunction with CWT based noise suppression as a tool to analyze the time-frequency characteristics of shock and vibration and compare its analytical effectiveness to conventional CWT based time-frequency analysis. In a controlled laboratory setting, acceleration of wooden pallets associated with shock and vibration was measured. Results of the analysis show that the process of frequency reallocation through synchrosqueezing employed by SST can better resolve joint frequency and time resolution at CWT. In conclusion, it can be stated that SST is an effective tool for modeling and simulation of non-stationary signals such as shock and vibration.

References

[1] Kipp, W., 1998: *PSD and SRS in Simple Terms*. ISTA Conference, Orlando, FL.

[2] Pittner, S., and Kamarthi, S. *Feature Extraction for Wavelet Coefficients for Pattern Recognition Tasks*. IEEE Transactions on Pattern Analysis and Machine Learning. 1999. 21 (1) 83-88.

[3] Usner, M., and Aldroubi, A. *A Review of Wavelets in Biomedical Applications*. Proceedings of IEEE. 1986. 84 (4) 626-638.

[4] Choudhary, D., Malasri, S., Harvey, M., and Smith, A. *Time-Frequency Analysis of Shock and Vibration Measurements Using Wavelet Transforms*. International Journal of Advanced Packaging Technology. 2014. 2 (1) 60-69.

[5] Hollowell, B., and Smith, S. *A Proposed Method to Standardize Shock Response Spectrum (SRS) Analysis*. International Environmental Science and Technology Journal. 1996. 39 (3) 19-24.

[6] Daubechies, I., Lu, J., and Wu, H-T. *Synchrosqueezed Wavelet Transforms: An Empirical Mode Decomposition-Like Tool.* Appl. Comput. Harmon. Anal. 2011. 30; 243-261.

[7] Mallat, S., 2009: *A Wavelet Tour of Signal Processing: A Sparse Way*. 3rd Ed. Academic Press, 832.

[8] Lewalle, J., and Keller, D., 2005: *Analysis of Web Defects by Correlating 1-D Morlet and 2-D Mexican Hat Wavelet Transforms*. Proc. of SPIE, Wavelet Applications in Industrial Processing III, 63-74, Boston, MA.

[9] Latsenko, D. *Linear and Synchrosqueezed Time–Frequency Representations Revisited: Overview, Standards of use, Resolution, Reconstruction, Concentration, and Algorithms*. Digital Signal Processing. 2015. 42; 1-26.

[10] Herrera, R., Han, J., and Baan, M. *Applications of the Synchrosqueezing Transform in Seismic Time Frequency Analysis*. Geophysics. 2014. 79 (3) V55-V64.

[11] Donnoho, D., and Johnstone. *Ideal Spatial Adaptation by Wavelet Shrinkage*. Biometrika. 1994. 81 (3) 425-455.

[12] Brevdo, E., Jang, D., Oh, H., and Kim, D. *SynchWave: Synchrosqueezed Wavelet Transform*. 2013. https://cran.r-project.org/web/packages/SynchWave/index.html