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Time-Frequency Analysis of Shock and Vibration Measurements Using Wavelet Transforms

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Abstract This article presents the use of the Continuous Wavelet Transform (CWT) for the analysis of shock and vibration measurements. Acceleration measurements from pallets dropped from five different heights and vibration measurements of pallets are acquired in controlled laboratory settings. Power spectral density (PSD) as estimated from CWT is compared to the Shock Response Spectrum as well as the PSD estimated from Fourier Transform (FT) and Short Time Fourier Transform (STFT). CWT overcomes the drawbacks of Fourier Transform in analyzing non-stationary signals such as shock and vibration data. CWT also provides more improved time-frequency resolution than STFT. The article presents results that indicate that CWT can be used as an effective spectral analysis tool for shock and vibration measurements.

Keywords *Continuous Wavelet Transform; Fourier Transform; Shock Response Spectrum; Vibration*

1. Introduction

Packaged products often undergo shock and vibration during distribution. An accurate simulation of the shock and vibration phenomenon enables effective testing of packaging components and provides direction for further improvement of packaging and transportation design. For this purpose, understanding the spectral (frequency) components that are present as a result of stimulus caused by shock and vibration is important. A commonly used spectral analysis tool in the area of signal processing is the Power Spectral Density (PSD). Conventional PSD is computed using the Fourier Transform (FT) which assumes that any signal is composed of a weighted summation of sinusoids of various frequencies [1]. For the signal being analyzed, the PSD represents the power associated with each of these sinusoids. The draw back in the use of PSD based on FT is its inability to accurately represent signals that are non-stationary [2]. Non-stationary signals contain different frequency components at different periods of time. Shock is a transient event defined as a mechanical disturbance characterized by a rise and decay of acceleration in a short period of time, while vibrations are random oscillations about a reference point, usually for a longer period of time [3]. Given the definitions and observations made through measurements, shock and vibration are considered non-stationary processes. Short Time Fourier Transform (STFT) attempts to address non-stationarity by estimating the spectral content of the signal over small segments of the signals using a sliding window. However, the time-frequency uncertainty principle limits the accuracy of STFT. The Shock Response Spectrum (SRS) is another approach to analyze shock data that assumes a model containing a set of single degree-of-freedom, mass-damper-spring oscillator subsystems that are excited by base motion [4]. For each subsystem, the natural frequency and maximum amplitude of response is determined [3]. The plot of maximum amplitude versus natural frequency is the SRS. Although originally developed for transients associated with shock, SRS is also used for analysis of vibration [5]. Wavelet Transform maps a temporal signal on to a 3-D time-frequency space and is used extensively to analyze non stationary signals [6, 7]. In this article, a technique applying Continuous Wavelet Transform (CWT) is used for spectral analysis of shock and vibration. The technique measures the PSD based on the CWT coefficients. CWT accounts for the non-stationary properties of shock and vibration by not only computing the frequency components present in the signal, but it also computes the time intervals when those frequencies are present. The time-frequency localization properties of wavelet basis functions in conjunction with the mechanism of the transform process, makes CWT an extremely effective spectral analysis tool.

The subsequent sections in this article are as follows: In section 2, data collection methods, signal processing algorithms and software tools are described. Section 3 discusses the results of the analysis of the data and section 4 presents the conclusions drawn from this research.

2. Materials and Methods

In this section, first, a description of the shock and vibration experiment is provided. Next, the signal processing techniques including FT, STFT, CWT, PSD and SRS for analyzing the data are presented. Finally, the software tools to implement the analysis are discussed.

2.1. Data Collection Procedure

For recording shock data, a Lansmont Saver 3M30 recorder was used to measure acceleration versus time at 1000 samples/sec along three directions. It was attached to a pallet, which was raised and dropped from a certain height. In this experiment, a wooden pallet was dropped from 2 inches, 4 inches, 6 inches, 8 inches and 10 inches. Figure 1 shows the setup of the shock experiment. For each height, acceleration versus time was measured through the three channels of the shock recorder. Channel 3 measured the acceleration along the direction of the drop, while the other two channels measuring acceleration along the other two orthogonal directions. For measuring vibrational data, a wooden pallet was mounted on a vibration platform as shown in Figure 2. The Lansmont recorder was used to measure the vibrational acceleration versus time signal sampled at 1000 samples/sec along three orthogonal directions (x, y and z axis). A truck vibration simulation in accordance with ASTM D 4169 Truck Level I was utilized.

Figure 1: *Experimental Setup for Collection of Shock Data*

Figure 2: *Experimental Setup for Collection of Vibration data*

2.2. Signal Processing/Modeling Techniques

In this sub-section, the theoretical background and software tools to compute FT, STFT, CWT as well as the calculation of PSD for each of the stated signal processing technique are presented. In the context of this article, the time varying function $x(t)$ represents the acceleration versus time signal associated with the shock or vibration data. Further, the signal $x(t)$ is normalized by subtracting its mean value from the signal. The mean value corresponds to the zero frequency or the DC component. Thus the normalization prevents the possibility of the zero frequency component from dominating the PSD plots shown in this article. This typically improves clarity of the figures without loss of relevant information.

2.2.1. Fourier Transform

Fourier Transform of a temporal signal $x(t)$ is given by [8]: $X(f)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$, where $\omega = 2\pi f$ and *j* is the complex number $\sqrt{-1}$. $X(f)$ is the representation of the signal $x(t)$ in the Fourier or frequency domain. Fourier transform expresses the signal $x(t)$ as a weighted sum of the basis function: $e^{-j\omega t} = cos(\omega t) + jsin(\omega t)$. The equation can be interpreted as follows. Fourier transform in essence decomposes the signal $x(t)$ into constituent sinusoids and the transform finds amplitude and phases of these constituent sinusoids. For a specific value of ω, the signal $x(t)$ is correlated with the basis function: $cos(\omega t) + jsin(\omega t)$. The complex correlation coefficient obtained for that value of ω is the corresponding Fourier Transform coefficient. The complex coefficient represents the amplitude and phase of the sinusoid of frequency ω. This process is repeated for values of ω ranging from $-\infty$ to ∞.

2.2.2. Short Time Fourier Transform

The Short Time Fourier Transform (STFT) is a modification of the conventional Fourier Transform. In STFT, the time domain signal, $x(t)$, is broken into segments. Fourier Transform of each of these segments is the STFT. The process of dividing $x(t)$ into segments is achieved by multiplying the signal with a sliding window function $g(t-\tau)$. The parameter ~~τ~~ τ controls the shift or the slide of the window *g(t)* . In this research, a Hanning window of size 10 was used to represent *g(t)*.

STFT of a signal $x(t)$ is given by [9]: $X(\tau,\omega) = \int_{-\infty}^{\infty} x(t)g(t-\tau)e^{-j\omega t}dt$, where $\omega = 2\pi f$ is frequency in radians/second. The plot of STFT coefficients for the signal $x(t)$ is a 3D plot with the x-axis representing the time shift ~~τ~~ τ and frequency represented on the y-axis. The amplitude of the STFT coefficients is represented on the z-axis.

2.2.3. Continuous Wavelet Transform

Wavelet Transform represents a signal $x(t)$ as a weighted sum of basis functions referred to as wavelets. The weights correspond to the wavelet coefficients. The Continuous Wavelet Transform (CWT) of a signal $x(t)$ is given by [10]: $X(\tau,s) = \int x(t)\frac{1}{\sqrt{s}}\phi^*\left(\frac{t-\tau}{s}\right)$, where τ is the translation parameter and s is the scale parameter. The basis function $\phi(t)$ is referred to as a mother wavelet. $\phi^*(t)$ is the complex conjugate of $\phi(t)$. The translation parameter, τ , shifts $\phi(t)$ in time and the scale parameter, *s,* controls the temporal width of $\phi(t)$. The scale parameter is inversely related to frequency. An example of a mother wavelet function is a Morlet function. The Morlet wavelet is a complex valued function given by: $\phi(t) = e^{-2\pi^2\frac{t^2}{z_0^2}}\left(e^{j2\pi t} - e^{2\pi^2 t^2}\right)$. The envelope factor z_o controls the number of oscillations in the wavelet with a typical value of $z_o = 5$ [11]. The Morlet basis function is used in this article for the computation of CWT.

The CWT, in simpler terms, is the correlation of the signal $x(t)$ with various shifted and stretched/shrunken versions of the mother wavelet $\phi(t)$. It is this ability to manipulate the width (stretching or shrinking) of the mother wavelet and shift it along the time axis that makes the CWT time-frequency analysis effective. The plot of CWT coefficients for the signal $x(t)$ is a 3D plot. The x-axis corresponds to the time shift, ~~τ~~ τ. The y-axis represents frequency *f* or scale *s*. The amplitude of the CWT coefficients is represented by the z-axis.

2.2.4. Power Spectral Density

Power Spectral Density (PSD) of a signal represents the distribution of power over various frequencies that compose the signal. It is the average or expected value of the Fourier Transform of the signal $x(t)$ computed over an infinite time period. PSD of a signal $x(t)$ is given by:

$$S_x(f) = \lim_{T\to\infty} E\left\{\frac{1}{2T}|X(f)|^2\right\} = \lim_{T\to\infty} E\left\{\frac{1}{2T}\left|\int_{-T}^{T} x(t)e^{-jwt}\,dt\right|^2\right\}$$

T refers to the period over which the statistical average *E{}* of the Fourier Transform, *X(f),* is computed*.* The above equation can be implemented using computer algorithms based on techniques such as the Welch's Method. Welch's method computes the PSD of a digitized signal $x[i]$ using the following steps [12]:

- Partition the signal $x[i]$ in K overlapping segments, each of length *L,* with M points overlapping between adjacent segments.

- Next, each segment, $x_k[i]$, is multiplied by a window function W and the modified periodogram is computed using an N-point Discrete Fourier Transform (DFT) as shown in equation below.

$$A_k(n) = \frac{1}{L}\sum_{i=0}^{L-1} x_k(i)W(i)e^{\frac{-2\pi in}{L}}$$

Here *n* = 0, 1, 2, ... *N*-1 and *k* = 0, 1, 2, ... *K*-1. *N* is the number DFT points and *K* is the number of segments used in partitioning the data, $x[i]$. A particular value of *n* corresponds to a frequency $f_n = \frac{nf_s}{N}$, where f_s is the sampling frequency of the signal. An example of a window function is:

$$W(i) = 1 - \left[\frac{i - \frac{L-1}{2}}{\frac{L+1}{2}}\right]^2$$

- The PSD is then estimated using the equation

$$\hat{S}_x(f_n) = \frac{L}{UK}\sum_{k=1}^{K}|A_k(n)|^2 ,$$

Where, $U = \frac{1}{L}\sum_{i=0}^{L-1} W^2(i)$

The implementation of DFT is commonly done using the Fast Fourier Transform. PSD based on STFT and CWT is estimated by simply computing the magnitude squared of the respective transform coefficients. In this article, with regard to the PSD plots shown in the results section, FT based plots are 2-D figures with PSD on the y-axis and frequency on the x-axis. PSD from STFT and

CWT are 3-D figures with PSD on the z-axis, while time and frequency are on the x and y axes respectively.

2.2.5. Shock Response Curve

In order to compute the Shock Response Curve (SRS), it is assumed that the system is composed of a set of single degree-of-freedom oscillator subsystems. Each subsystem has its own frequency response that peaks at its natural frequency. The acceleration versus time data measured from a shock or vibration experimentation is then filtered by using the frequency response of the SDOF subsystems. The maximum amplitude at the output of the filtering processing for each SDOF subsystem is noted. SRS is a plot of the maximum amplitude versus the natural frequency of each of the SDOF subsystem [13].

2.3. Software Analysis Tools

The shock and vibration data collected in the experiments are processed using Matlab programming language to compute PSD from FT using Welch’s Method, STFT and CWT. SRS was computed using software developed by Tom Irvine based on the Kelly Richman algorithm [14, 15].

3. Results and Discussion

Figure 3 shows the acceleration, PSDs and SRS for the shock experiment for a 2 inch drop along the direction of the drop (z axis). Figure 4 shows the acceleration, PSDs and SRS for the vibration experiment along the z axis. Figures 3 and 4 are exemplar plots and the observations derived through these are consistent for other measurements from the experiment as well. A caveat needs to be pointed out about SRS. SRS is calculated by computing the maximum amplitude of the response for a set of SDOF oscillators. Hence, the motivation and mathematical background associated with SRS is different from that of other signal processing techniques used in this article. In comparing the various techniques, it can be observed that PSD from FT and SRS are 2-D plots that represent frequency domain information about the signal and do not capture temporal information. The PSD from STFT and CWT are 3-D plots that capture both temporal and frequency domain information. All the techniques identify the dominant frequency at about 75 Hz. For FT based PSD and SRS, there is no information about the time periods when these frequencies are present. Both STFT and CWT indicate that the dominant frequency component occurs approximately in a temporal region around 0.2 seconds. Uncertainty principles in time-frequency resolution dictates that the time instant when a specific frequency signal occurred can only be estimated up to certain accuracy. This means that temporal accuracy is always gained at the cost of losing frequency localization and vice versa. STFT shown in Figure 3 was computed with high temporal resolution. As a result, the frequency resolution of STFT is compromised and this is represented by the exaggerated presence of frequency components in the 100 Hz - 200Hz range. If STFT were to be computed with emphasis on frequency resolution, the temporal resolution would be lost and localization along the temporal axis would deteriorate. On the other hand, by controlling the scale and the shift parameters of the wavelet basis function for computing the transform, PSD from CWT innately balances both temporal and frequency resolutions. This is apparent from the CWT PSD in Figure 3, which shows better localization along the time and frequency axis around 0.2 seconds and 75Hz (the dominant frequency). The presence of these frequencies is also noticed, with lower power, around 0.3 seconds in both CWT and STFT. Similarly, inferences can be made for vibrational data analysis as represented in Figure 4. The vibrational data analysis shows strong frequency components in the frequency band less than 100 Hz. CWT and STFT show that these frequency components occur around 0.1 seconds and 0.2 seconds. Given the advantages in terms of time-frequency representations of STFT and CWT, both techniques, however, are computationally challenging when compared to FT. An alternative to CWT is the discrete wavelet transform (DWT). It should be noted that the disadvantage with DWT is that, in order to achieve computational efficiency, DWT uses truly discrete time and frequency locations in its computations by algorithmically skipping certain locations on the time-frequency plots. This makes DWT plots less intuitive for visualization and interpretation in its raw form.

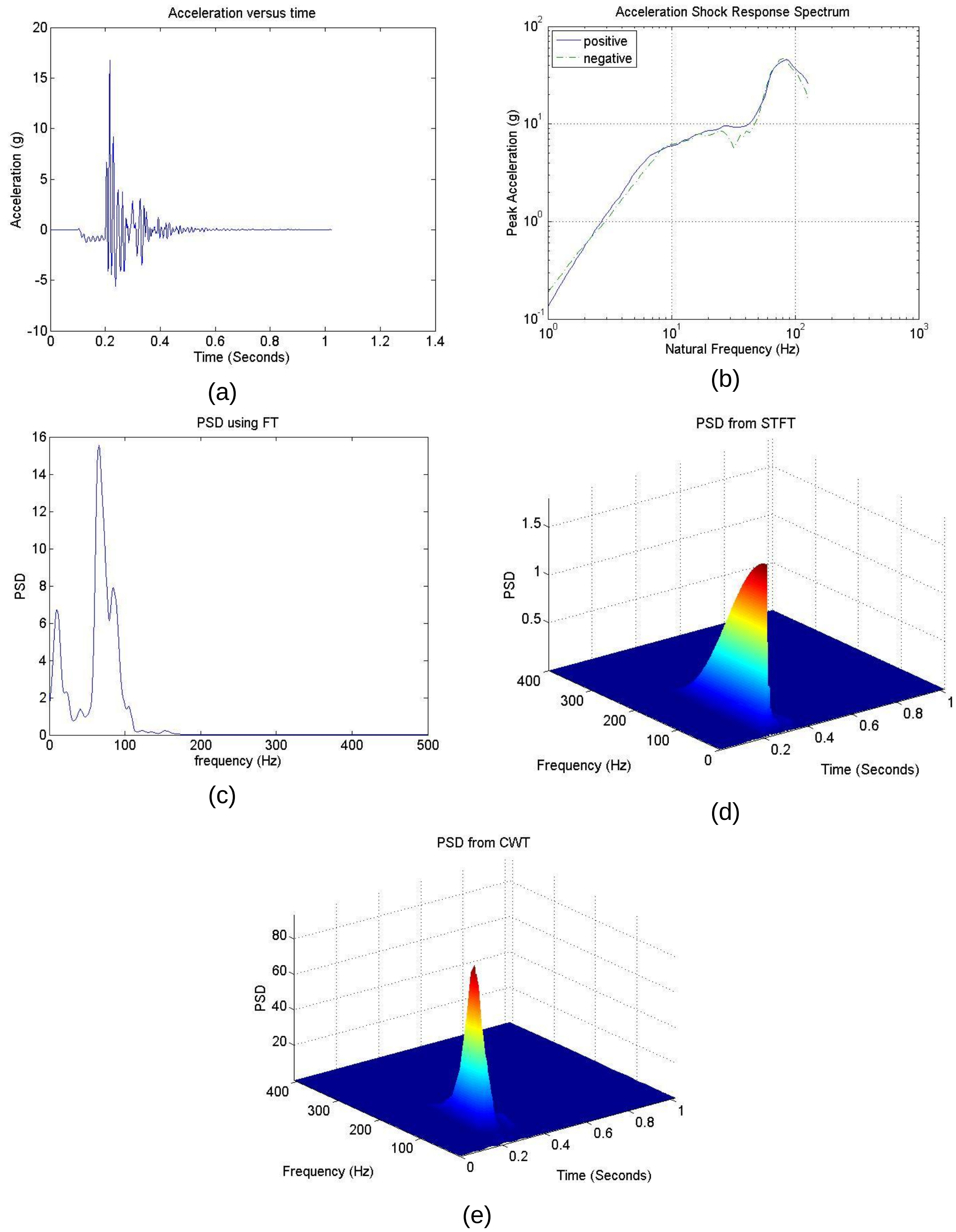

***Figure 3**: Data Measurement and Analysis for 2 Inch Pallet Drop Measured in the Direction of the Drop. Shock Acceleration versus Time Measurement (a), SRS (b), PSD from FT (c), PSD from STFT (d), and PSD from CWT (e)*

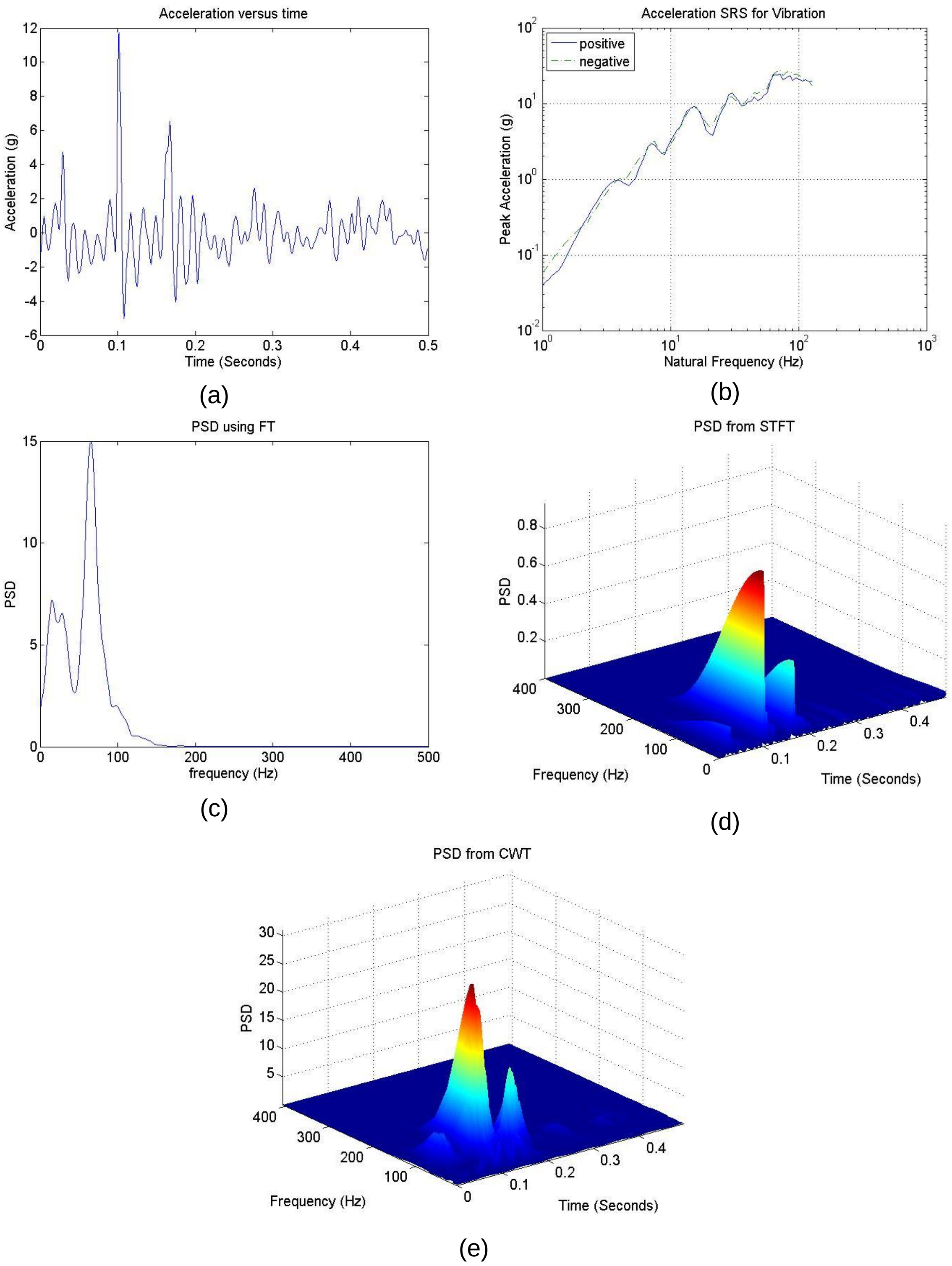

Figure 4: *Data Measurement and Analysis of Vibration along the Z-Axis. Vibrational Acceleration Versus Time Measurement (a), SRS (b), PSD from FT (c), PSD from STFT (d), PSD from CWT (e)*

4. Conclusion

The focus of this article was to present CWT as a tool to analyze the time-frequency characteristics of shock and vibration and compare its analytical effectiveness to conventional techniques such as

SRS and PSD based on FT. In a controlled laboratory setting, acceleration of wooden pallets associated with shock and vibration was measured. PSD based on FT, STFT and CWT was computed. SRS was also calculated from the shock and vibrational data. Results of the analysis show that CWT has the ability to provide optimum joint frequency and time resolution. In using STFT there is a tradeoff between temporal and frequency resolutions. FT provides solely frequency domain representation of the signal with no information about time periods when the frequency components occur. SRS on the other hand provides a plot of maximum amplitude response versus natural frequencies by assuming a set of subsystems with SDOF. This article concludes that with the ability to present both time and frequency information with optimum localization, CWT is an effective tool for modeling non-stationary signals such as shock and vibration.

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