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Analysis of Electromagnetic Field Using FEM: A Review

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Abstract Before designing any electrical device or apparatus, it is imperative to ascertain the eddy current losses that would occur as per a given design specification. Pre-determining the losses before the design, would expedite the design process besides making it easier to implement the post corrections in the design criteria. The numerical technique harnessed here is the finite element method, which has been used to compute the eddy current losses in different magnetic structures. This method is gaining grounds these days, with the advancement in the technological scenario, witnessed so far. It has been able to fruitfully address the problems encountered in the arena of eddy currents. The bottlenecks or discrepancies experienced in the earlier methods have been eradicated by harnessing the power of this numerical approach. Using the tools of FEM, one can expect higher levels of accuracy and easier set up of design model. It incorporates the complex boundary constraints with the careful combination of the flexibility and ease of programming in the use of algebraic equations.
Keywords *Finite Element Method (FEM), Method of Moments (MOM), Boundary Element Method (BEM), Electromagnetic Field, Numerical Methods*

1. Overview

The understanding and contemplation of any natural phenomena be it in the form of biological, mechanical or electrical necessitates sound knowledge of physics, the laws of which govern all these occurrences in nature. The laws of physics constitute equations (algebraic, differential or integral) which are the different types of formulations that can be obtained by representing physical quantities as variables. The solutions of these equations are a pre-requisite for investigating the behaviour of these phenomena. For the analysis and design of electric devices, familiarity with electromagnetic fields generated therein is not only helpful but also mandatory. The knowledge of field distribution helps to gain information about the symmetry of the fields' w.r.t. a particular reference. The investigation of the fields requires the exploration of different methods available for electromagnetic field problems. The section that follows gives a bird's eye view of the different methods available for the investigation of electromagnetic field problems.

2. Review of the Different Formulations for Electromagnetic Problems

There are different approaches that have been applied in the recent past for formulating numerous electromagnetic problems. As the number of these problems is varied, there can be umpteen types of approaches that have been found to give reasonable results since their time of occurrence. Below gives a glimpse of the finite number of techniques for dealing with these problems.

2.1. Analogue Procedures

The procedure rests its foundation on obtaining the unknown field by experimental measurements on an analogue of the field region i.e., on a field region governed by the same equations and with the same boundary and interface conditions as that of the problem region. These procedures have generally been used only by Laplace's equations under two or three dimensional conditions. In fact, it has been practically impossible to model inhomogeneties, non-linear ties and henceforth. Besides, in the three dimensional version (such as electrolytic tank, or resistive network), this method is rather expensive and cumbersome. The more convenient versions (for instance, graphite paper or elastic membrane) are restricted to two dimensional fields.

2.2. Analytical Methods

These methods compute the value of the unknown quantities by adopting analysis of the problem using various techniques. The development of analytical methods advanced a good deal while numerical methods were still in their infancy. These methods are still widely used and include series solution and conformal mapping techniques. The first attempt of electromagnetic field problems was characterized by the analytical treatment of Maxwell's equations (1).

These methods are still widely used and include series solutions and conformal mapping techniques. Series solutions are generally obtained by the so called method of separation of variables. The method can be mainly applied to Lapalce's equation or Hemholtz equation in two or three dimensional problems and can also be applied to time dependent problems.

Conditions The use of this method requires the fulfillment of the following conditions:
a. Every boundary or interface must coincide with an equi-coordinate surface.
b. The coordinate system must allow the separation of variables.

Both these conditions are rarely satisfied in a complex problem. The calculations require the use of special functions (such as Bessel, Legendre, or Elliptic) which often are not so easy to handle.

Conformal Mapping Approach The solution using this particular technique is widely used. It is based on the properties of analytic complex functions and can be applied to problems, reducible to Laplace's equations in two dimensional regions.

Limitations
a. It suffers from constraints governed by the boundary and interface conditions.
b. There is lack of generality in these kinds of problems.
c. A lot of effort goes in obtaining the field solutions and developing special algorithms while making use of this method.
d. They are capable of solving only a limited class of problems involving basic homogeneous and linear media with simple geometric configurations.

2.3. Graphical Methods

These methods analyze a given problem by making use of different tables and charts like the transmission line charts, smith charts etc. The variations of the problem are mapped into these charts and the corresponding observations are noted. Their use is restricted to Laplace's equations for two dimensional geometries as they are generally based on the properties of analytic functions.

Limitation Their accuracy is limited even when they are carefully applied.

2.4. Numerical Methods

The impediments experienced in the afore-mentioned methods have given the way for the drift towards the numerical methods. Whenever exact analytical solutions are not available, approximate methods are sought. There are several numerical techniques that can be used to overcome the drawbacks of the analytical methods. Electromagnetic field calculation has been revolutionized over the last three decades by the rapid development of the digital computer with progressively greater capacity and speed and a decreasing cost of arithmetic operations.

Any numerical field computation involves, among others, the following steps:

a. Definition of the region over which the governing equation is to be solved. This is the approximate problem domain (discretization of the field region).
b. Formulation of the discrete problem.
c. Derivation of a system of equations whose solution gives the numerically computed approximate field (1).

They can be broadly sub-divided into the following categories:

a. Finite difference method
b. Boundary element method
c. Finite element method

These methods, as their names suggest, vary greatly in the behavior as also their manner of implementation. These are discussed in the sections that follow:

A*. Finite Difference Method* It is one of the older and yet decreasingly used numerical methods. In essence, it consists in superimposing a grid on the space-time domain of the problem and assigning discrete values of the unknown field quantities at the nodes of the grid. Then, the governing equation of the system is replaced by a set of finite difference equations relating the value of the field variable at a node to the value at the neighboring nodes.

Limitation
a. Lack of geometrical flexibility in fitting irregular boundary shapes.
b.Large points are needed in regions where the field quantities change very rapidly.
c.The treatment of singular points and boundary interfaces that do not coincide with constant coordinate surfaces (1, 2).

B*. Boundary Element Method* To formulate the eddy-current problem as a boundary element technique, an integral needs to be taken at the boundary points. To avoid the singularity which occurs in the integrand when the field point corresponds to the source point, the volume is enlarged by a very small hemisphere whose radius tends to zero, with the boundary point being the center of the sphere. The usage of the boundary element method reduces the dimensionality of the problem from three to two or from two to one. It is found to be useful in open boundary problems where it strongly challenges the finite element method.

Limitation Instead of sparse (and usually symmetric and positive definite) matrices of the FDM and FEM, the resultant matrices in this method are full (and usually non-symmetric) (1).

C. Finite Element Method The most powerful numerical method appears to be the FEM, which from the mathematical point of view can be considered as an extension of the Rayleigh-Ritz / Galerkin technique of constructing coordinate functions whose linear combination approximate the unknown solutions. In this method, the field region is subdivided into elements i.e. into sub-regions where the unknown quantities , such as , for instance, a scalar or vector potential , are represented by suitable interpolation functions that contain, as unknowns, the values of the potential at the respective nodes of each element. The potential values at the nodes can be determined by direct or iterative methods.

The normal procedure in a field computation by the FEM involves, basically, the following steps:

a. Discretization of the field region into a number of node points and finite elements.
b. Derivation of the element equation: - The unknown field quantity is represented within each element as a linear combination of the shape functions of the element and in the entire domain as a linear combination of the basis functions. A relationship involving the unknown field quantity at the nodal points are then obtained from the problem formulation for a typical element. The accuracy of the approximation can be improved either by subdividing the region in a finer way or by using higher order elements (1).
c. Assembly of element equations to obtain the equations of the overall system. The imposition of the boundary conditions lead to the final system of equations, which is then solved by iterative or elimination methods.
d. Post-processing of the Results: - To compute other desired quantities and to represent the results in tabular form or graphical form, etc.

Limitation
a. As the number of elements is increased, a colossal amount of time is required for the execution of the program.
b. The time required to prepare the input data and to interpret the result is also considerable.

This numerical technique can be applied to a wide class of problems. In general, by increasing the number of elements, improved results are obtained. In order to reduce computational and manpower costs, self-adaptive finite element mesh generators have been developed and are widely used today. These generators produce mesh structures from the outline of the problem with the minimum of user-specified information and lead to an acceptable accuracy of the resulting solution in the minimum time.

3. Framework of the Finite Element Method

In the numerical analysis, the finite element method (FEM) is used for solving partial differential equations approximately. Solutions are approximated by either eliminating the differential equation completely (steady state problems), or rendering the PDE into an equivalent ordinary differential equation, which is then solved using standard techniques such as finite difference method. etc. The use of the finite element method in engineering for the analysis of physical systems is commonly known as finite element analysis.

The method was introduced by Richard Courant to model the effect of torsion on the shape of a cylinder. Courant's contribution was evolutionary, drawing on large earlier results for partial differential equations developed by Rayleigh, Ritz and Galerkin. Since now, much progress has been made in the development of the finite element method for the analysis of electromagnetic problems, especially in the following five areas:

a. The first is the development of higher-order vector finite elements, which make it possible to obtain highly accurate and efficient solutions of vector wave equations.
b. The second is the development of perfectly matched layers as an absorbing boundary condition. Although the perfectly matched layers were intended primarily for the time-domain finite-difference method, they have also found applications in the finite element simulations.
c. The third is perhaps the development of hybrid techniques that combine the finite element and asymptotic methods for the analysis of large, complex problems those were unsolvable in the past.
d. The fourth is the further development of the finite element--boundary integral methods that incorporate fast integral solvers, such as the fast multi-pole method, to reduce the computational complexity associated with the boundary integral part.
e. The last, but not the least, is the development of the finite element method in time domain for transient analysis. As a result of all these efforts, the finite element method has gained more popularity in the computational electromagnetic community and has become one of the preeminent simulation techniques for electromagnetic problems.

Below mentioned problems encompass the different types of electromagnetic problems that are of interest to the design engineer:

a. Static field problems
b. Quasi-static field problems
c. Eddy current problems
d. Transient field problems

These problems can be formulated using Maxwell's equations and need to satisfy certain constraints. These problems along with their constraints are expressed in the form of partial differential equations. There are three stages in the process of FEM:

a. Pre-processor:- This stage, in essence, prepares the problem to be formulated in terms of known variables, finite nodes and their corresponding nodal equations. At the end of this stage, the problem is ready to be processed.
b. Processor Stage: - This stage involves the computation of field variables at the respective nodes by employing various available processing techniques. The techniques at the disposal are presently the different software packages like MATLAB, FORTRAN.
c. Post-processing Stage: - The results obtained with the processing stage are now compared with analytical results, obtained as a result of empirical formulas. After this, the error analysis is done at this juncture. The error analysis postulates to some extent, the conformity of the obtained results with the desired ones (3).

4. Formulation of the Finite Element Method

The FEM is concerned with the solution of mathematical or physical problems which are generally defined in a continuous domain either by local differential equations or by equivalent global statements. To render the problem amenable to numerical treatment, the infinite degrees of freedom of the system are discretized or replaced by a finite number of unknown parameters, as indeed is the practice in other processes of approximation. The concept of 'finite elements' replaces the continuum by a number of sub-domains (or elements) whose behaviour is modeled adequately by a limited number of degrees of freedom and which are assembled by processes well known in the analysis of discrete systems. Hence this method can be defined as any approximation process in which:

a. The behaviour of the whole system is approximated by a finite number n of parameters aj, i = 1 to n. These parameters are described by "n" number of equations.
b. The "n" equations governing the behaviour of the whole system

$$Fj\ (ai) = 0 \qquad j = 1 \text{ to } n \tag{1.1}$$

can be assembled by the simple process of addition of terms contributed from all sub-domains (or elements). These elements divide the system into physically identifiable entities (without overlap or exclusion). Then

$$Fj = \Sigma\ Fej \tag{1.2}$$

where F^e_j is the element contribution to the quantity under consideration

This method combines the best of the features found in the earlier used methods like the variational method, Rayleigh Ritz method, and so forth. The implementation of this method involves steps in the following chronological order:

4.1. Discretization of the Continuum

The electromagnetic field is described as a continuum of numerous points. The field variable is projected as having been endowed with infinite degrees of freedom, as it can be expressed as a function of different coordinates of each point in the solution domain. The finite element method aims to approximate this field to finite degrees of freedom. By thus transforming this problem into finiteness, the finite element method thus divides the solution region into known number of non-overlapping sub-regions or elements. Thereafter, nodes are assigned to different elements.

4.2. Selecting Approximating or Interpolation Function

Within each element, an approximation for the variation of potential is sought which is described by an interpolation function. This function inter-relates the potential distribution in various elements such that the potential is continuous across inter-element boundaries. Now, the field variable may take any one of the form from vector, scalar or a tensor. Depending on its form, the corresponding variation of the potential is approximated and hence the choices of a particular interpolation function. More often, polynomial functions are used as interpolation function for the ease of their differentiability as well as integrability. The potential, in general, is non-zero within an element and zero outside its periphery. The element shape functions are denoted by αi and have the following properties:

$$\alpha i\ (xi, yi) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{1.3}$$

4.3. Element Governing Equations

On the completion of the above two steps, equations describing the properties of elements are derived for different elements. These equations are then combined to form the element coefficient matrices. For each element, a typical element coefficient matrix is obtained. This computed value of this matrix when, viewed as a determinant, gives the numerical value of the area of that particular element. The value of the matrix is found to be positive if the nodes are numbered counterclockwise (starting from any node).

4.4. Assembling All Elements

Having derived matrix for individual elements, the next step is to assemble all such elements in the solution region. The basic idea behind this is to obtain the overall or global coefficient matrix, which is the amalgam of individual coefficient matrices.

4.5. Imposition of Boundary Constraints

Before going for the solution of the global coefficient matrix, it is mandatory to impose certain boundary constraints. Keeping this in view, these matrices are modified accordingly. For obtaining a unique solution of the problem, two possibilities can be examined:

a. In some cases, a value of potential is assigned across a line. If the specified potential is same everywhere, equi-potential conditions are said to be specified. When the potential is set to zero, this condition is termed as Dirichlet homogeneous condition.
b. In others, a value of the normal derivative of the potential is specified. When this value is set to be zero, this is known as Neumann homogeneous condition.

4.6. Solving the Resulting Equations

The matrix equations so obtained after accounting for the boundary constraints, are then solved, using a suitable procedure. The task behind obtaining the solution of the equations is to compute the value of field variable at the nodes. That is to find the variation of the field variable within each node.

4.7. Error Analysis

The results obtained above are compared with standard results in order to obtain extent of conformity with the desired ones. The desired or standard results are acquired from the empirical formulas. Thereafter, the error analysis is carried out. In case, the error is found to be exceeding the required tolerance limits, the results obtained from the equations are again channeled through the iterative procedure. At this juncture, the power of this numerical approach can be realized. Iterative techniques tend to refine the results, every time a process does not conform to the desired level of accuracy (3, 4, 5).

5. Solution Approach for Finite Element Method

The finite element formulation can be dividing according to the procedure by which the equations for the nodal values are formulated. The basic approaches are discussed below:

5.1. Direct Method

This involves substituting the nodal values (obtained from above) into the governing equations to finally obtain the nodal value equations. For employing the finite element method, the problem is formulated in terms of partial differential equation. But when making use of this method, the problem must be formulated in the integral form.

5.2. Variational Method

In the variational formulation, the differential equation of the physical system is formulated in terms of an equivalent energy functional, which in some applications may represent the stored energy or the dissipated power in the system. This functional usually has the property of being stationery about the correct set of functions representing the required solution of the electromagnetic problem, subject to the relevant boundary constraints. The Euler equation of this functional shall generally coincide with the original partial differential equation. Thus, the well known method of calculus of variations can be applied in solving the variational problem whose Euler equation is identical to the differential equation of the system. In eddy –current problem, the energy functional can be expressed in terms of either the basic field quantities E and H alone or the vector and scalar potentials (1).

5.3. Galerkin Weighed Residual Method

Some cases of electromagnetic field analysis witness the absence of a variational expression. There, this method can be fruitfully applied. Using this approach, the governing equation of the problem is stated without relying on the functional or variational statement. Its inherent ability to extend the FEM approach to problems having non-existence of functional gives it a definite edge over others. In this approach, a trial function, that satisfies all the boundary conditions, is selected. This function is said to approximately satisfy the system equations too. When this trial function is substituted into the system equations, it amounts to results with some error in the form of residuals. It is observed that closer the approximation, the faster the residual converges to zero (6, 7).

6. Interpolation Functions

In the period of early development of the FEM, in the 1950’s, the selection of behaviour functions for the various types of elements under development was a haphazard affair. Functions were often chosen to correspond to an intuitive feeling for the response of the class of the problem to be modeled. The conditions which all functions ought to meet were not identified, nor were organized schemes to establish functions for any desired geometric form and number of node points (8).

6.1. Polynomial Functions

These give a powerful means for the description of complex behaviour and simultaneously, lend themselves well to the processes of integration and differentiation. In one dimension, the general expression can be written as:

$$\Phi = a_1 + xa_2 + x^2a_3 + \dots + x^na_n \qquad (1.4)$$

The prime requirement is the fulfillment of continuity of the field variable. This must be able to satisfy all the conditions. If the field variable is found to be continuous at the interface of the element, zero order continuity is obtained. In case, first derivatives are also continuous then first order continuity is obtained and henceforth (8, 9, 10).

7. Comparison between FDM and FEM

There are three different ways by which a physical problem can be mathematically formulated:

a. By an appropriate partial differential equation
b. By applying a variational principle or a weak integral form over the problem domain.
c. By utilizing a proper integral equation.

Historically, the above analytical formulations have led to the development of three apparently unrelated, numerical discretization procedures. These processes are FDM, FEM and MOM respectively.

7.1. Finite Difference Method

This method is perhaps the easiest and the most traditional method commonly used in EM problems. It is essentially a discretization procedure whereby the differential equation is describing a given problem is converted into a finite difference equation. The regularity of the mesh size makes the computer program easier to construct. The solution using this approach contains an immense amount of numerical information from which it is possible to obtain some useful quantities of interest. A major drawback is that it attempts too much by filling all space with its net (11, 12, 13).

7.2. Finite Element Method

The possibility of a completely free topology for a mesh is introduced using this technique which is based on the energy distribution rather than on the differential equation describing the equilibrium condition. This approach is characterized by two distinctive features:

a. The domain of the problem is viewed as a collection of non-intersecting simple sub-domains called finite elements.
b. Over each finite element, the solution is approximated by a linear combination of undetermined parameters and pre-selected algebraic polynomials.

Beyond these two features, FEM is a variational method, like the Rayleigh Ritz and Galerkin methods in which the approximate solution is sought in the form

$$u \approx \sum_{n=1}^{N} a_n \phi_n \qquad (1.5)$$

where "Φn" are pre-selected functions called shape functions and
"a_n" are parameters that are determined using an equivalent to the governing equation u.

8. Conclusion

A major drawback of traditional variational methods is the selection of approximation functions for geometrically complex domains and boundary conditions. This impediment is overcome in FEM by representing a geometrically complex domain as a collection of sub-domains that allow an easy construction of approximation functions. The difference between different finite element approaches lie in the choice of shape functions. Finite element analysis can easily handle an inhomogeneous domain. The disadvantage of the method lies in the irregularity of the meshes which requires additional effort in preparing the input data. Since the finite element computational meshes are rectangular in shape, they do not conform to curved surfaces, as in the case of cylindrical boundary. The use of conforming meshes can overcome this problem. Although FDM yields point approximation to differential equations, FEM integrates point approximation through space to yield area averaged approximations.

There is general disagreement about the advantages or disadvantages of using FDM in preference to FEM and vice-versa. However, it is generally felt that for irregular domains, FEM is often easier to use and that for regular domains FDM is more easily programmed. The choice between FDM and FEM in field computation is finely balanced, depending on experience, skill in programming, computer software availability and storage capacity, and of course the problem which is being investigated (14, 15, 16).

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