

A Study of Scheduling Algorithms to Maintain Small Overflow Probability in Cellular Networks with a Single Cell

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Publication Date: 9 February 2016

Article Link: <http://technical.cloud-journals.com/index.php/IJACSIT/article/view/Tech-488>



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Abstract Wireless scheduling algorithms for the download of a single cell that can maximize the asymptotic decay rate of the queue-overflow probability as the overflow threshold approaches infinity. We first derive an upper bound on the decay rate of the queue-overflow probability over all scheduling policies. Specifically, we focus on the class of “ α - algorithms,” the base station picks the user for service at each time that has the largest product of the transmission rate multiplied by the backlog raised to the power α . The α -algorithms arbitrarily achieve the highest decay rate of the queue-overflow probability. We design a scheduling algorithm that is both close to optimal in terms of the asymptotic decay rate of the overflow probability and to maintain small queue-overflow probabilities over queue-length ranges of practical interest.

Keywords *Asymptotically Optimal Algorithms; Cellular System; Large Deviations; Queue-Overflow Probability; Wireless Scheduling*

1. Introduction

Link scheduling is an important functionality in wireless networks due to both the shared nature of the wireless medium and the variations of the wireless channel over time. In the past, it has been demonstrated that by carefully choosing the scheduling decision based on the channel state and/or the demand of the users, the system performance can be substantially improved [2].

As per the survey, most of the scheduling algorithms focus on stable throughput to the users. Consider a cellular network with a single cell. The base-station transmits to users. There is a queue Q_i associated with each user $i=1,2,\dots,N$.

Due to interference, at any given time, the base-station can only serve the queue of one user (refer Figure 1 and Figure 2). Hence, this system can be modeled as a single server serving N queues.

Assume that data for user arrives at the base-station at a constant rate λ_i . Furthermore, assume a slotted model, and in each time-slot the wireless channel can be in one of states.

In each state, if the base-station picks user i to serve, the corresponding service rate is F_m^i . Hence, at each time-slot Q_i , increases by λ_i , and if it is served and the channel is at state, Q_i decreases by F_m^i . We assume that perfect channel information is available at the base-station. In a stability problem, the goal is to find algorithms for scheduling the transmissions such that the queues are stabilized at given offered loads.

For a given $\alpha \geq 1$ if the channel is in stable state, the base-station chooses the user with the largest $(Q_i)^\alpha F_m^i$. delay-sensitive applications; it is far from sufficient [1]. In this paper, we are interested in the probability of queue overflow, which is equivalent to the delay-violation probability under certain conditions. The question that we attempt to answer is the following: Is there an optimal algorithm in the sense that, at any given offered load, the algorithm can achieve the smallest probability that any queue overflows, i.e., the smallest value of $P[\max_{1 \leq i \leq N} Q_i(T) \geq B]$. Note that if we impose a quality-of-service (QoS) constraint on each user in the form of an upper bound on the queue-overflow probability, then the above optimality condition will also imply that the algorithm can support the largest set of offered loads subject to the QoS constraint.

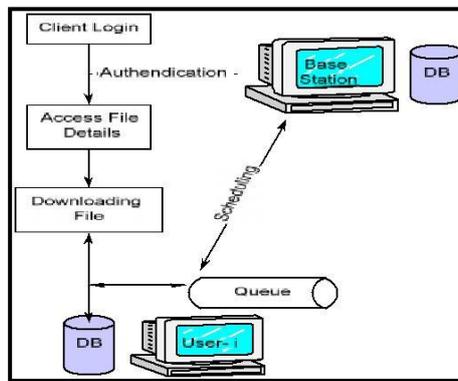


Figure 1: Service Provided Using Queues

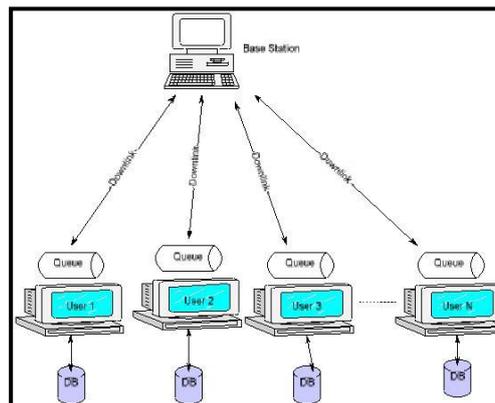


Figure 2: Dataflow Diagram

To emphasize the dependency on α , in the sequel we will refer to this class of throughput-optimal algorithms [3] as α -algorithms. While stability is an important first-order metric of success, for many We use large-deviation theory and reformulate the QoS constraint in terms of the asymptotic decay rate of the queue-overflow probability as B approaches infinity. In other words we are interested in finding scheduling algorithms that can achieve the possible value of

$$\liminf_{B \rightarrow \infty} 1/B \log P \left[\max_{1 \leq i \leq N} Q_i(0) \geq B \right]$$

Our main results are the following. We show that there exists an optimal decay rate I_{opt} such that for any scheduling algorithm

$$\liminf_{B \rightarrow \infty} 1/B \log(P \left[\max_{1 \leq i \leq N} Q_i(0) \geq B \right]) \geq -I_{opt}.$$

Furthermore, for α -algorithms

$$\liminf_{B \rightarrow \infty} 1/B \log(P^\alpha \left[\max_{1 \leq i \leq N} Q_i(t) \geq B \right]) \leq -I_{opt}.$$

For the above problem, it is natural to use the large-deviation theory because the overflow probability that we are interested in is typically very small. Large-deviation theory has been successfully applied to wire line networks and to wireless scheduling algorithms that only use the channel state to make the scheduling decisions.

The α -algorithms encounters a Multidimensional calculus-of-variations (CoV) problem for finding the “most probable path to overflow.” The decay rate of the queue-overflow probability then corresponds to the cost of this path, which is referred to as the “minimum cost to overflow.” Unfortunately, for many queue-length-based scheduling algorithms of interest, this multidimensional calculus-of-variations problem is very difficult to solve. In the literature, only some restricted cases have been solved: Either restricted problem structures are assumed (e.g., symmetric users and ON–OFF channels), or the size of the system is very small (only two users).

In this paper, we use Lyapunov function to overcome the difficulty of the multidimensional CoV problem.

The “exponential rule” can maximize the decay rate of the queue-overflow probability over all scheduling policies. The results in this paper are comparable but different. The advantage of working with the algorithms instead of the exponential rule is that the algorithms are scale-invariant (i.e., the outcome of the scheduling decision does not change if all queue lengths are multiplied by a common factor). Hence, we can use the standard sample-path large-deviation principle (LDP) instead of the refined LDP used is more technically involved.

In addition, our results highlight the role that the exponent plays in determining the asymptotic decay rate. Finally, using the insight of our main result, we design a scheduling algorithm that is both close to optimal in terms of the asymptotic decay rate of the overflow probability and empirically shown to maintain small queue-overflow probabilities over queue-length ranges of practical interest.

2. An Upper Bound on the Decay Rate of the Overflow Probability

We first present an upper bound I_{opt} on $I_0(\lambda)$ under a given offered load λ .

This value I_{opt} bounds from above the decay rate for the overflow probability of the stationary backlog process $Q(t)$ overall scheduling policies. Then we conclude that the upper bound on the decay rate overflow rate is

$$\liminf_{B \rightarrow \infty} 1/B \log(P \left[\max_{1 \leq i \leq N} Q_i(0) \geq B \right]) \geq -I_{opt}.$$

3. A Lower Bound on the Decay Rate of the Overflow

3.1. Probability for the α -Algorithms

To provide a lower bound that relates the decay rate of the probability to the “minimum cost to overflow” among all fluid path. For ease of exposition, instead of considering the stationary system, we consider a system that starts at time 0 (although the results can also be extended to the stationary system).

We will use the following modified queue-overflow event: $\{V_\alpha(qB(t)) \geq 1\}$,

Where $V_\alpha(q) \triangleq \left(\sum_{i=1}^N (qi)^{\alpha+1} \right)$. It turns out that computing the large-deviation decay rate requires solving a CoV problem that is very difficult. The reason to use the modified overflow metric $V_\alpha(q^B(t))$ is that the corresponding decay rate is much easier to compute and $V_\alpha(q^B(t))$ approximates the function when α is large. The function $V_\alpha(Q)$ is a Lyapunov function for the α -algorithm. Hence, this function may be viewed as throughput-optimal algorithm; the exponential rule is not scale-invariant.

4. Asymptotical Optimality of α -Algorithms

We will establish that in the limit as $\alpha \rightarrow \infty$, the α -algorithms asymptotically achieve the largest minimum cost to overflow equal to I_{opt} .

To emphasize the dependence on α we use the probability distribution conditioned on $Q(0)=0$ under the α -algorithm.

$$\liminf_{B \rightarrow \infty} 1/B \log(P^\alpha \left[\max_{1 \leq i \leq N} Q_i(t) \geq B \right]) \leq -I_{opt}.$$

As $\alpha \rightarrow \infty$, we would expect that the α -algorithm would give more and more preference to the link with the largest queue backlog among all links with nonzero rates. If there are several links that have the same (largest) backlog, the link with the highest rate among them would be served. However, we caution that if we choose $\alpha = \infty$, then the resulting algorithm is the max-queue algorithm, which is not throughput-optimal for general channel models. Therefore, the above intuition does not directly lead to a stable scheduling policy.

5. Conclusion

In this paper, we study wireless scheduling algorithms for the downlink of a single cell that can maximize the asymptotic decay rate of the queue-overflow probability as the overflow. Specifically, we focus on the class of “ α -algorithms,” the base station picks the user for service at each time that has the largest product of the transmission rate multiplied by the backlog raised to the power α . A key step in proving this result is to use a Lyapunov function to derive a simple lower bound for the minimum cost to overflow under “ α -algorithms”. This technique, overcomes the multidimensional calculus-of-variations problem. Finally, using the insight from this result, we design hybrid scheduling algorithms that are both close to optimal in terms of the asymptotic decay rate of the overflow probability and empirically shown to maintain small queue-overflow probabilities over queue-length ranges of practical interest. For future work, we plan to extend the results to more general network and channel models.

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