Active Force on Retaining Wall Supporting Φ Backfill Considering Curvilinear Rupture Surface

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Abstract The evaluation of active earth pressure coefficient for safe design of retaining wall constructed to retain the supporting material at different elevation on two sides is one of the most important parameter. In this paper an effort has been made to provide an analytical expression for static active earth pressure acting on inclined rigid retaining wall considering non-linear failure surface by applying the horizontal slice method and limit equilibrium principle which gives more general solution compared to linear kind of failure surface. Considering curvilinear rupture surface the effect of wide range of parameters like angle of internal friction (Φ), angle of wall friction (δ), wall inclination angle (α), surcharge loading (q) are taken in to account to evaluate the static active earth pressure co-efficient. The results are presented in terms of static active earth pressure co-efficient and compared with the available solutions.

Keywords Active Earth Pressure, Φ Backfill, Rigid Retaining Wall, Wall Inclination, Curvilinear Rupture Surface

1. Introduction

Earth pressure theories constitute one of the most important parts in the civil engineering structures. Active earth pressure theories have been analyzed by our prominent researchers in the most accurate ways. Coulomb (1776) [1] was the first to establish the formulation for active and passive earth pressure for the retaining structures. Rankine’s theory (1857) [2] has been a landmark for determination of active and passive earth pressure. Graphical methods have also been introduced by Culmann (1865) [3]. All these analyses have been conducted considering the Φ nature of backfill. In most of the cases, the determination of active earth pressure coefficient developed by several researchers based on the assumption that the failure surface is to be planner. In this analysis, considering non-linear failure surface and applying Horizontal Slice Method of analysis the optimum value of static active earth pressure coefficient is calculated.
2. Method of Analysis

A retaining wall of height, H inclined at an angle α with the vertical as shown in the figure 1 has been considered with the failure surface being curvilinear. The failure surface makes angles of θ_n and θ_1 with the vertical at bottom and top respectively. The failure wedge is split into ‘n’ number of thin slices of thickness ΔH. The rate of change of failure angle (θ_1 and θ_n) has been assumed as \( θ_R = \frac{(θ_1 - θ_n)}{(n-1)} \). Free Body Diagram of retaining wall-backfill system under active state of equilibrium is shown in figure 2.

The forces acting on the wall has been calculated by considering the following parameters:

- \( H_{i,1}, H_i = \) Horizontal shear acting on the top and bottom of the \( i^{th} \) slice
- \( W_i = \) Weight of the failure wedge of \( i^{th} \) slice
- \( V_{i,1}, V_i = \) Vertical load (UDL) on top and bottom of \( i^{th} \) slice
- \( Φ = \) the angle of internal friction of soil
- \( P_{ai} = \) Active earth pressure on \( i^{th} \) slice
- \( R_i = \) the reaction of the retained soil on \( i^{th} \) slice
- \( δ = \) the angle of wall friction

3. Derivation of Formulations Considering Active State of Equilibrium

Applying the force equilibrium conditions for \( i^{th} \) slice, we can solve the equations in the following pattern:

\[
\sum H = 0
\]

\[
P_i \cos(δ + α) = R_i \cos(φ + δ_1 + (i-1)δ_r)
\]

\[
-γ(ΔH)^2 \tan φ \left( \sum_{m=i}^{n-1} \tan(θ_i + mδ_1) + (n-i)\tan α - (i-1)\tan(θ_i + (i-1)δ_r) + \tan α \right)
\]

\[
\sum V = 0
\]

\[
P_i \sin(δ + α) = -R_i \sin(φ + δ_1 + (i-1)δ_r) + (i-\frac{1}{2})γ(ΔH)^2 (\tan α + \tan(θ_i + (i-1)δ_r))
\]

Solving the above equations, the generalized equation is derived as follows:

\[
P_{ai} = \frac{γ}{2}(ΔH)^2 \left[ \frac{(2i-1)(\tan α + \tan(θ_i + (i-1)δ_r))\cos(φ + δ_1 + (i-1)δ_r)}{(i-1)(\tan(θ_i + (i-1)δ_r) + \tan α)\sin(φ + δ + α + δ_1 + (i-1)δ_r)} \right]
\]

Where, \( \tan(θ_i + mδ_1) = 0 \) for \( i = n \)

The active earth pressure coefficient can be simplified as,
Optimisation of the active earth pressure coefficient $K_a$ is done for the variables $\theta_1$ and $\theta_n$ satisfying the optimization criteria. The optimum value of $K_a$ is given in table 1.

**4. Parametric Study**

A detailed Parametric study has been conducted considering non-linear failure surface to find the effect of variations of wide range of parameters like soil friction angle ($\Phi$), wall inclination ($\alpha$), wall friction angle ($\delta$) on evaluation of static active earth pressure coefficient for $\Phi = 10^0, 20^0, 30^0, 40^0$; $\delta = 0, \Phi/2, \Phi$ and $\alpha = +20^0, 0, -20^0$. The details of these studies are presented below:

4.1. Effect of Inclination of the Wall ($\alpha$)

The effect of inclination of the wall on the evaluation of static active earth pressure for different value of wall inclination angle $\alpha$ is shown in figures 3 to 5. From these plots, it is seen that the magnitude of static active earth pressure increases with the increase in wall inclination angle $\alpha$. The reason behind it is that when the inclination of the wall is positive with the vertical then it has to support more soil in comparison to the wall when it is inclined negative with the vertical. For example at $\Phi = 20^0$ and $\delta = 0^0$, the increase in $K_a$ is 48% for $\alpha = +20^0$ over $\alpha = 0$ value, whereas the decrease in $K_a$ is 21.2% over $\alpha = 0$ value for $\alpha = -20$. Again at $\Phi = 30^0$ and $\delta = \Phi/2$, the increase in $K_a$ is 89% for $\alpha = +20^0$ over $\alpha = 0$ value, whereas the decrease in $K_a$ is 41.09% over $\alpha = 0$ value for $\alpha = -20$. Again at $\Phi = 20^0$ and $\delta = \Phi$, the increase in $K_a$ is 60% for $\alpha = +20^0$ over $\alpha = 0$ value, whereas the decrease in $K_a$ is 28.9% over $\alpha = 0$ value for $\alpha = -20$. It is also observed that the value of $K_a$ for $\alpha = +20^0$ and $\delta = \Phi$, decreases with the increase in the value of $\Phi$ upto $\Phi = 20^0$, then the value suddenly increases with the increase in the value of $\Phi$. 

![Figure 1: Inclined Retaining Wall (Active state of equilibrium)](image)
Figure 2: Detailed drawing showing various components of the retaining wall along with slices (active state of equilibrium)
4.2. Effect of Wall Friction Angle (δ)

The effect of wall friction angle on the evaluation of static active earth pressure coefficients for different value of wall friction angle δ is shown in figures 6 to 8. From these plots, it is seen that the magnitude of active earth pressure coefficient is going to be decreased due to increase in δ value. The reason behind it is that the frictional resistance of wall and soil is increasing with the increase in the value of δ. For example at Φ = 10° and α =20°, the decrease in K_a is 2.7% for δ = Φ/2 over δ = 0 value, whereas the decrease in K_a is 3.6% for δ = Φ over δ = 0 value. Again at Φ = 20° and α =0, the decrease in K_a is 8.5% for δ = Φ/2 over δ = 0 value, whereas the decrease in K_a is 11.6% for δ = Φ over δ = 0. Again at Φ = 30° and α =-20°, the decrease in K_a is 17.5% for δ = Φ/2 over δ = 0 value, whereas the decrease in K_a is 20.3% for δ = 2Φ/3 over δ = 0. It is also observed that the value of K_a for α = +20° and δ = Φ, decreases with the increase in the value of Φ upto Φ = 20°, then the value suddenly increases with the increase in the value of Φ.
4.3. Effect of Surcharge (q)

Figure 9 shows the variations of active earth pressure for inclusion of surcharge. It is seen that the value of active earth pressure increases gradually with the increase of surcharge. For $\Phi = 30^\circ$, $\alpha = 20^\circ$ and $\delta = \Phi/2$, the increment in $K_a$ is 28% and 55% for $q=10\text{KN/m}^2$ and $20\text{KN/m}^2$ respectively compared to $q = 0$ for a constant height. Also at $\Phi = 30^\circ$, the value of active earth pressure decreases compared to other $\Phi$ values.

Figure 9: Shows the variations of active earth pressure coefficient with respect to soil friction angle ($\Phi$) for different surcharge loads ($\alpha = 20^\circ$, $\delta = \Phi/2$)
4.4. Wall Inclination and Nonlinearity of Failure Surface

Figure 10 shows the nonlinearity of failure surface of backfill (active case) for different values of wall inclinations (α = -20°, 0°, +20°). The shape of the failure surface may be sagging or hogging in nature depending on the soil and wall properties. The shape of the failure surface is linear if the value of wall friction angle δ = 0 and the wall is vertical. With the increase in wall friction angle δ value the nonlinearity of the failure surface is increases. For example, at Φ=30°, δ= Φ/2 and α = +20°, the value of failure angle at bottom is 54° whereas the value of failure angle at top is -41°. Also figures 11-12 show that the failure wedge is quite different as compared to the failure surface of the Ghosh and Sengupta (2012) [4] analysis. It is seen that the failure angle reduces with the increase in the wall inclination angles. The comparison shows that the value of failure angle is 26° in case of Ghosh and Sengupta (2012) [4] for the aforesaid conditions.

**Figure 10:** Shows the nonlinearity of failure surface of backfill (active case) for different values of wall inclinations, α = -20°, 0°, +20° at Φ = 30°, δ= Φ/2

**Figure 11:** Shows the comparison between failure surface of backfill (active case) for wall inclination, α = +20° at Φ = 30°, δ= Φ/2
4.5. Comparison of Results

Figure 13 shows the variations of active earth pressure coefficient with respect to soil friction angle (Φ) at Wall friction angles δ= Φ/2 for α = 20°. K_a decreases uniformly with the increase in the value of soil friction angle (Φ). It can also be observed from table 2 that the value of K_a is around 5-15% higher than the values of Classical Coulomb (1776) theory.
Table 1: Active Earth Pressure Coefficients (Static Case)

<table>
<thead>
<tr>
<th>φ</th>
<th>δ</th>
<th>Co-efficient of Active Earth Pressure, ((K_a))</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>(\alpha = 20)</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0.841</td>
</tr>
<tr>
<td></td>
<td>(\phi/2)</td>
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</tr>
<tr>
<td></td>
<td>(\phi)</td>
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</tr>
<tr>
<td>20</td>
<td>0</td>
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</tr>
<tr>
<td></td>
<td>(\phi/2)</td>
<td>0.664</td>
</tr>
<tr>
<td></td>
<td>(\phi)</td>
<td>0.693</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>0.542</td>
</tr>
<tr>
<td></td>
<td>(\phi/2)</td>
<td>0.584</td>
</tr>
<tr>
<td></td>
<td>(\phi/3)</td>
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<td>(\phi)</td>
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<tr>
<td>40</td>
<td>0</td>
<td>0.436</td>
</tr>
<tr>
<td></td>
<td>(\phi/2)</td>
<td>0.638</td>
</tr>
<tr>
<td></td>
<td>(\phi)</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 2: Shows the comparison of results: -Active case

<table>
<thead>
<tr>
<th>φ</th>
<th>δ</th>
<th>α</th>
<th>Present Study</th>
<th>Coulomb (1776)</th>
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<tr>
<td>10</td>
<td>5</td>
<td>20</td>
<td>0.819</td>
<td>0.8013</td>
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<td>20</td>
<td>10</td>
<td>20</td>
<td>0.664</td>
<td>0.6147</td>
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<td>30</td>
<td>15</td>
<td>20</td>
<td>0.584</td>
<td>0.4762</td>
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<tr>
<td>40</td>
<td>20</td>
<td>20</td>
<td>0.638</td>
<td>0.3694</td>
</tr>
</tbody>
</table>

5. Conclusion

In this paper, an analytical solution has been developed to obtain active earth pressure coefficients using horizontal slice method. The analysis develops curvilinear rupture surface, may be sagging or hogging in nature depending upon the soil-wall parameters. The results obtained from the present solution are compared with the available solution like Coulomb (1776) method and it is concluded that the present solution gives higher value of active earth pressure co-efficient. The active force decreases with the increase in \(\phi\) and \(\delta\); whereas, it is also observed that the negative inclination of wall \((\alpha)\) and surcharge \((q)\) reduces the value of active earth pressure coefficient. The present analysis gives higher value of coefficient of active earth pressure in comparison to linear failure surface analysis. The present study can be further extended for determination of seismic active and passive earth pressure co-efficient considering non-linear nature of failure surface using both pseudo-static and pseudo-dynamic methods.
References


Notations

- $\theta_1$ = Failure surface angle with vertical for top slice
- $\theta_n$ = Failure surface angle with vertical for bottom slice
- $\theta_R$ = Rate of change of failure surface angle
- $\Phi$ = Soil friction angle
- $\delta$ = Wall friction angle
- $\alpha$ = Wall inclination angle with the vertical
- $P_a$ = active earth pressure
- $H_1, H_2$ = horizontal shear
- $\Delta H$ = height of each slice
- $W_i$ = weight of $i^{th}$ slice
- $R$ = soil reaction force
- $V_1$ = vertical load (UDL) acting on the bottom surface of the $1^{st}$ layer
- $V_2$ = vertical load (UDL) acting on the top surface of the $1^{st}$ layer
- $\gamma$ = unit weight of soil