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Computation of Passive Earth Pressure Coefficients for a Horizontal Cohesionless Backfill Using the Method of Slices

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Abstract An approximate and acceptable solution for a retaining wall problem is developed using the method of slices. The failure surface consisting of log spiral followed by a tangent is considered in the analysis. To make the analysis statically determinate, the effect of wall friction is assumed to decrease linearly away from the wall. Static equilibrium conditions are used for the analysis of forces acting on the slices. The passive pressure coefficients for a vertical retaining wall with different wall friction and internal friction angles are computed. The results show a close agreement with some of the available solutions.
Keywords *Cohesionless Soil, Horizontal Backfill, Log-Spiral, Method of Slices, Passive Earth Pressure Coefficient*

1. Introduction

Coulomb (1776) and Rankine (1857) assumed planar failure surface and proposed methods for the estimation of earth pressure on the retaining walls. Later Terzaghi (1943) proposed a failure mechanism in which, the failure surface consisted of a log spiral originating from the wall base, followed by a tangent, which met the ground surface at an angle corresponding to Rankine's (1857) passive state. Several research workers have adopted this mechanism.

Caquot & Kerisel (1948) and Kerisel & Absi (1990) proposed a log spiral mechanism and presented their results in the form of charts. Janbu (1957), Sheilds & Tolunay (1973), Basudhar & Madhav (1980), and Kumar & Subba Rao (1997) used the method of slices for computing passive pressure coefficients in respect to cohesion-less soil by considering soil mass in the state of limit equilibrium. Soubra & Macuh (2002) proposed a method based on rotational log-spiral failure mechanism with the upper-bound theorem of limit analysis for the computation of passive earth pressure coefficients. Lancellotta (2002) provided an analytical solution for the passive earth pressure coefficients based on the lower bound theorem of plasticity. Shiau et al., (2008) used an approach based on upper and lower bound theorems of limit analysis coupled with finite element formulation and nonlinear programming techniques for the analysis of passive earth pressures.

In the proposed analysis, the method of slices is employed for the computation of passive thrust for a vertical wall retaining horizontal cohesion-less backfill, using the failure mechanism suggested by Terzaghi (1943), i.e., a log spiral and its tangent.

2. Proposed Method

In the proposed method, the effect of wall fiction is assumed to decrease linearly away from the wall. The unique failure surface is fixed using the procedure suggested by Shield and Tolunay (1973). Static equilibrium equations are used for the analysis of forces acting on the slices. Figure 1 shows a vertical retaining wall AB, with a horizontal cohesion-less backfill. The failure surface consists of log spiral BC, that originates from the wall base, with tangent, CD meeting the backfill surface at an angle, (45^o - $\phi/2$), where ϕ is the angle of soil internal friction. At C, there is a conjugate failure plane CA, passing through the wall top. Thus, as seen from Figure 1, the pole of the log spiral lies on the line CA. The resultant of the normal and shear forces acting on the failure surface will pass through the pole, O, of the logarithmic spiral creating no unbalanced moments. Considering the fact that, the direction of the failure surface at the wall is dependent on the roughness angle δ, of the wall, Sheilds & Tolunay (1973) have developed an equation for α_w in terms of ϕ and δ, which represents the angle between the horizontal and the failure surface at the wall. It is considered positive when it is above horizontal and negative when it is below the horizontal.

$$\alpha_w = \frac{1}{2}\left[arc\cos\left[\cos(\phi-\delta)-\frac{\sin(\phi-\delta)}{\tan\phi}\right]-\phi-\delta\right] \quad (1)$$

Equation 1 can also be used to find inclination α_n for different values of ϕ and δ for the linear decrease of δ away from the wall.

From Figure 1, the following information is generated:

θ = maximum spiral angle
r_o = initial radius of log spiral at the wall base
r_1 = radius corresponding to the maximum spiral angle θ
B_R = width of the conjugate Rankine's failure wedge, AE
δ = wall friction angle
ϕ = soil internal friction angle
α_w = angle between the horizontal and the failure surface at the wall

Figure 1- Failure mechanism adopted in the proposed analysis
Figure 2 - Proposed Method of Slices

From Figure 2, the following information is generated:

H = height of the retaining wall, AB
H_R = height of the Rankine's wall, CE
x_i = distance of the left edge of i^{th} slice from point E
P_R = passive thrust acting on the Rankine wall, CE
P_p = passive thrust acting on the wall of height, AB=H

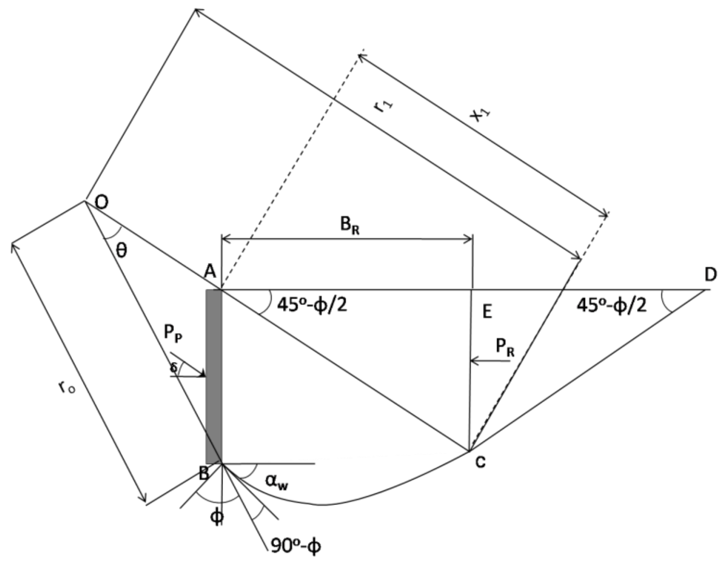

***Figure 1**: Failure Mechanism Adopted in the Proposed Analysis*

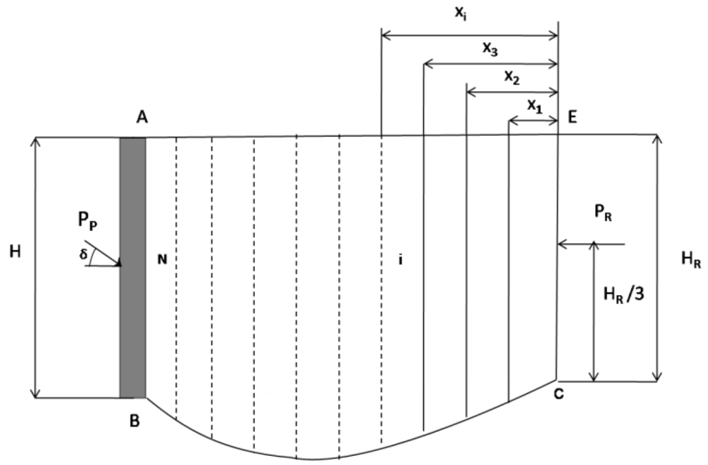

***Figure 2**: Proposed Method of Slices*

2.1. Geometry of the Failure Surface

Geometry of the failure surface is dependent on the values of soil internal friction ϕ and wall friction angle δ.

Referring to the Figure 1 and considering triangle, OAB,

$$\theta + \phi + \alpha_w + 180 - \left(45 + \frac{\phi}{2}\right) = 180 \qquad (2)$$

In the above equation, θ is given as

$$\theta = 45 - \alpha_w - \frac{\phi}{2}$$

and

$$\frac{r_o}{\sin\left(180-\left(45+\frac{\phi}{2}\right)\right)}=\frac{x_1}{\sin(\phi-\alpha_w)}=\frac{H}{\sin\theta} \quad (3)$$

Also from the equation of log spiral,

$$OC = r_1 = r_o.e^{\theta.\tan\phi}$$

and

$$CA = x_2 = r_1 - x_1 \quad (4)$$

Now considering triangle, CAE,

$$\frac{x_2}{\sin 90}=\frac{H_R}{\sin(45-\frac{\phi}{2})}=\frac{B_R}{\sin(45+\frac{\phi}{2})} \quad (5)$$

Considering N number of slices, the slice width b is calculated as

$$b=\frac{B_R}{N}$$

In order to make the analysis statically determinate, the variation of direction of the inter slice force P_{p_i} (Figure 3) is required to be assumed. With the assumption of linear variation of the inter slice force, δ_i (Figure 3) is expressed as,

$$\delta_i = \frac{x_i}{B_R}\delta, \text{where } x_i = ib$$

From the Rankine's (1857) mechanism,

$$P_R=\frac{1}{2}\gamma {H_R}^2\tan^2(45+\frac{\phi}{2}) \quad (6)$$

Figure 3: (a) Free body diagram of slice for i=1. (b) Free body diagram of the slices for i=1 to N

From Figure 3 (a) and Figure 3 (b) the following information is generated:

P_{p_1} = passive thrust acting on the first slice
b = width of the slice
W_1 = weight of the first slice
R_1 = resultant reaction acting at the base of the first slice
δ_1 = direction of the inter slice force for the first slice
α_1 = angle of inclination of the normal with the vertical on the base of the first slice
P_{p_i} = passive thrust acting on the i^{th} slice
W_i = weight of the i^{th} slice
R_i = resultant reaction acting at the base of the i^{th} slice
δ_i = direction of the inter slice force for the i^{th} slice
α_i = angle of inclination of the normal with the vertical on the base of the i^{th} slice

2.1.1. For the First Slice (i=1)

Referring to the Figure 3 (a) and considering the equilibrium of all the forces acting on the slice,

Horizontal force equilibrium

$$P_{p_1} \cos \delta_1 - P_R - R \sin(\phi + \alpha_1) = 0 \tag{7}$$

Vertical force equilibrium

$$\mathrm{R} \cos(\phi + \alpha_1) - \mathrm{P}_{p_1} \sin \delta_1 - \mathrm{W}_1 = 0 \tag{8}$$

Where,

$$\mathrm{W}_1 = \mathrm{bH_R} + \frac{1}{2}\mathrm{b}^2 \tan \alpha_1$$

From Equations 7 and 8,

$$\mathrm{P}_{p_1} = \frac{\mathrm{P_R} + \left(\mathrm{bH_R} + \frac{1}{2}\mathrm{b}^2 \tan \alpha_1\right) \tan(\phi + \alpha_1)}{[\cos \delta_1 - \sin \delta_1 \tan(\phi + \alpha_1)} \tag{9}$$

2.1.2. For Slices i=2 to N

Referring to the Figure 3 (b) and considering the equilibrium of all the forces acting on the slice,

Horizontal force equilibrium

$$\mathrm{P}_{p_i} \cos \mathrm{P}_{p_{i-1}} \cos \delta_{i-1} - \mathrm{R_i} \sin(\phi + \alpha_i) = 0 \tag{10}$$

Vertical force equilibrium

$$\mathrm{R_i} \cos\ (\phi + \alpha_i) - \mathrm{P_i} \sin \delta_i + \mathrm{P}_{p_{i-1}} \sin \delta_{i-1} - \mathrm{W_i} = 0 \tag{11}$$

Where,

$$\mathrm{W_i} = \mathrm{b}\left[\mathrm{H_R} + \mathrm{b}\left[\sum_{i=1}^{N} \tan \alpha_i\right]\right] + \frac{1}{2}\mathrm{b}^2 \tan \alpha_i$$

From Equations 10 and 11

$$\mathrm{P}_{p_i} = \frac{\mathrm{P}_{p_{i-1}} \cos \delta_{i-1} - \mathrm{P}_{p_{i-1}} \sin\delta_{i-1} \tan(\phi + \alpha_i) + \mathrm{W_i} \tan\ (\phi + \alpha_i)}{\cos \delta_i - \sin \delta_i \tan\ (\phi + \alpha_i)} \tag{12}$$

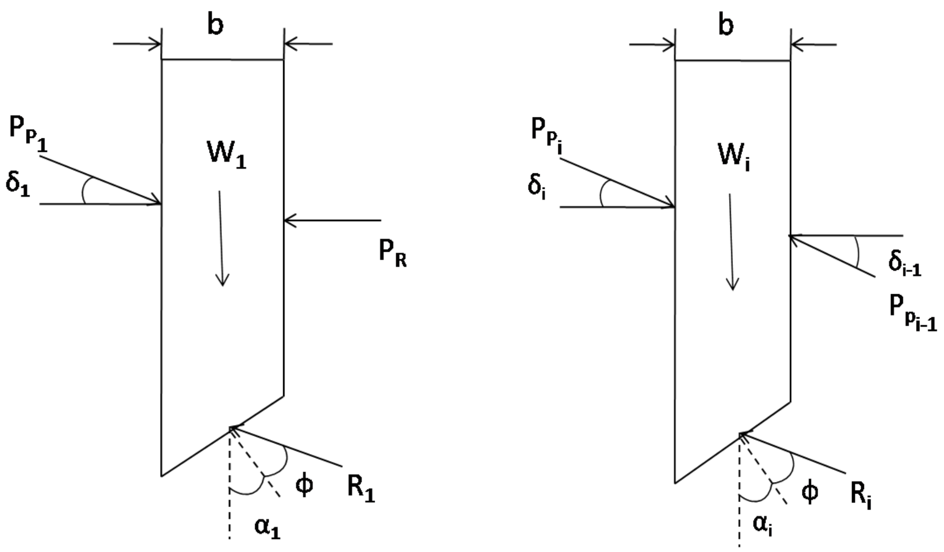

***Figure 3**: (a) Free Body Diagram of Slice for i=1. (b) Free Body Diagram of the Slices for i=1 to N*

2.2. Determination of Passive Earth Pressure Coefficient K_P

The soil wedge CDE (Figure 1), is in a passive Rankine (1857) state of equilibrium and the magnitude of passive force P_R acting on the wall CE is computed using Equation 6. In order to compute P_P, the soil wedge ABCE is divided in to N number of slices of width b and the analysis is carried out by considering all the forces acting on each slice as shown in the Figures 3(a) and (b). The passive thrusts are computed using equations 6, 9 and 12 for values of δ/ϕ from 0.1 to 1.0.

The passive earth pressure coefficient; K_P is then expressed as

$$K_P = \frac{2P_{P_N}}{\gamma H^2} = \frac{2P_P}{\gamma H^2}$$

3. Results and Discussion

The basic purpose of this analysis was to compute passive earth pressure coefficient, K_P and study its variation with respect to the parameters involved in the analysis. It was found convenient to express the value wall friction angle δ in terms of its ratio with respect to soil internal friction, ϕ of the soil, in a non-dimensional form (δ/ϕ).

***Table 1**: Passive Earth Pressure Coefficients for Different Combinations of ϕ andδ/ϕ*

δ/ϕ	K_P Values for the Corresponding ϕ Values				
	20	25	30	35	40
0	2.04	2.46	3.00	3.69	4.60
0.1	2.14	2.64	3.28	4.15	5.34
0.2	2.25	2.82	3.60	4.67	6.22
0.3	2.36	3.02	3.93	5.25	7.26
0.4	2.48	3.23	4.31	5.93	8.50
0.5	2.61	3.46	4.73	6.71	10.00
0.6	2.75	3.72	5.21	7.62	11.82
0.7	2.91	4.01	5.77	8.72	14.10
0.8	3.06	4.30	6.31	9.79	16.29

0.9	3.22	4.61	6.92	11.01	18.84
1	3.40	4.95	7.58	12.33	21.64

In Table 1, the values of passive earth pressure coefficient, K_P are shown for various combinations of non-dimensional ratio, δ/ϕ and soil internal frictional angle ϕ.

3.1. Comparison with other Solutions

In Table 2, computed values of K_P for ϕ = 20°, 30° and 40° and $\delta = \phi$ and $\phi/2$ are compared with other available solutions and in Table 3, percentage variations in the results obtained by the proposed method in comparison with other solutions are reported.

The values computed by Coulomb's (1776) theory are higher than the values obtained by the proposed method. For $\phi \leq 30^o$ and $\delta \leq \phi/2$, they are in the range 0 to 5.29%, and for values $\phi \geq 40^o$ and $\delta > \phi/2$, the variation is very high.

***Table 2**: Comparison of K_P Values*

Parameters	Passive Earth Pressure Coefficient, K_P					
Angle of soil friction, ϕ (degrees)	20		30		40	
Angle of wall friction, δ (degrees)	½ ϕ	ϕ	½ ϕ	ϕ	½ ϕ	ϕ
Proposed Method	2.61	3.4	4.73	7.58	10.00	21.64
Coulomb (1776)	2.89	3.53	4.98	10.1	11.77	92.57
Caquot & Kerisel (1948)	2.60	3.01	4.50	6.42	10.36	17.5
Janbu (1957)	2.60	3.00	4.50	6.00	9.00	14.0
Sokolovski (1965)	2.55	3.04	4.62	6.55	9.69	18.2
Shield & Tolunay (1974)	2.43	2.70	4.13	5.02	7.86	11.00
Chen (1975)	2.58	3.14	4.71	7.11	10.07	20.90
Basudhar & Madhav (1980)	2.56	3.12	4.64	6.93	9.56	19.35
Kumar & Subba Rao (1997)	2.5	3.07	4.6	6.68	9.8	18.86
William Powarie (1997)	2.52	2.87	4.4	5.8	8.92	14.32
Soubra & Macuh (2002)	2.57	3.13	4.65	6.93	9.81	20.1
Lancellotta (2002)	2.48	2.70	4.29	5.03	8.38	11.03
Shiau et al. (2008) lower bound	2.50	3.02	4.38	6.58	8.79	18.64
Shiau et al. (2008) upper bound	2.62	3.21	4.46	7.14	10.03	20.10
Kame et al. (2011) Kötter's Equation	2.97	3.29	5.13	6.57	10.1	16.46

The values reported by Chen (1975) are based on the limit analysis. These values are lower than the proposed values in the range 0 to 3.42%.

The values reported by Caquot & Kerisel (1948) are based on the limit equilibrium of a log spiral mechanism. For $\phi = 20^0$ to 40^0 and the $\delta = 0$ to ϕ, values of K_P as computed by them are lesser than the proposed values in the range 0 to 19.13%.

The values reported by Kumar & Subba Rao (1997) are based on the method of slices and are lesser than the proposed values. For $\phi = 20^0$ to 40^0 and $\delta = 0$ to $\phi/2$ the difference is very less and is in the range 0 to 2.0%. For $\delta > \phi/2$ the difference is relatively higher and is in the range 2.0 to 12.85%. Soubra & Macuh (2002) used the rotational log spiral failure mechanism with an upper-bound theorem of limit analysis. The values obtained from their analysis are lower than the proposed values in the range 0 to 7.12%.

With the analytical solution based on the lower bound theorem of plasticity, the K_P values as reported by Lancellotta (2002) are lower than the proposed values in the range 0 to 47.78%.

The K_P values reported by Shiau et al (2008) using lower bound theorem coupled with finite element formulations of limit analysis and nonlinear programming techniques are lower than the proposed values in the range 4.21 to 13.86%. The values obtained using upper bound theorem as compared to the proposed values are in the range +0.38 to -7.12%.

Kame et al., (2011) used Kötter's equation and the values obtained by them are higher than the proposed value initially for lower values of δ and for higher values of δ the values reported by them are lesser than the proposed values. They are in the range +27.45 to -23.94%.

The values of K_P reported by Janbu (1957) based on the limit equilibrium analysis. These values when compared with the proposed values are in the range 7.85 to -35.3%.

Similarly, the K_P value reported by Shields and Tolunay (1973) based on the limit equilibrium analysis are lower than the proposed values in the range, 0 to 49.17%.

***Table 3**: Comparison K_P Values*

Angle of Friction (degrees)		**Passive Earth Pressure Coefficient K_P**										
		Coulomb (1776)		Caquot & Kerisel (1948)		Kumar & Subba Rao (1997)		Soubra & Macuh (2002)		Lancellotta (2002)		Proposed method
Soil ϕ	Wall δ	K_P	% diff.	K_P	% diff.	K_P	% diff.	K_P	% diff.	K_P	% diff.	K_P
20		2.04	0	2.04	0	2.04	0	2.04	0	2.04	0	2.04
25		2.46	0	2.46	0	2.46	0	2.46	0	2.46	0	2.46
30	0	3.00	0	3.03	0	3.00	0	3.00	0	3.00	0	3.00
35		3.69	0	3.69	0	3.69	0	3.69	0	3.69	0	3.69
40		4.6	0	4.59	0	4.6	0	4.6	0	4.6	0	4.60
20		2.41	0.42	2.35	-2.08	2.38	-0.83	2.39	-0.42	2.35	-2.08	2.40
25		3.12	1.3	3.03	-1.62	3.06	-0.65	3.07	-0.32	3.07	-0.32	3.08
30	1/3 ϕ	4.14	2.22	4.00	-1.23	4.02	-0.74	4.03	-0.49	4.03	-0.49	4.05
35		5.68	4.03	5.28	-3.3	5.42	-0.73	5.44	-0.37	5.44	-0.37	5.46
40		8.15	6.96	7.25	-4.86	7.58	-0.52	7.62	0	7.62	0	7.62
20		2.64	1.15	2.6	-0.38	2.5	-4.21	2.57	-1.53	2.48	-4.98	2.61
25		3.55	2.6	3.4	-1.73	3.4	-1.73	3.41	-1.45	3.22	-6.94	3.46
30	1/2 ϕ	4.98	5.29	4.5	-4.86	4.6	-2.75	4.65	-1.69	4.29	-9.3	4.73
35		7.36	9.69	6.0	-10.58	6.6	-1.64	6.59	-1.79	5.88	-12.37	6.71
40		11.8	18	9.0	-10	9.8	-2.00	9.81	-1.9	8.38	-16.2	10.00
20		2.89	1.4	2.65	-7.02	2.73	-4.21	2.75	-3.51	2.58	-9.47	2.85
25		4.08	4.35	3.56	-8.95	3.72	-4.86	3.76	-3.84	3.41	-12.79	3.91
30	2/3 ϕ	6.11	9.69	5.00	-10.23	5.26	-5.57	5.34	-4.13	4.63	-16.88	5.57
35		9.96	19.71	7.1	-14.66	7.78	-6.49	7.95	-4.45	6.51	-21.75	8.32
40		18.7	40.92	10.7	-19.37	12.24	-8.06	12.6	-5.05	9.57	-27.88	13.27

20		3.53	3.82	3.01	-11.47	3.07	-9.71	3.13	-7.94	2.7	-20.59	3.4
25		5.6	13.13	4.29	-13.33	4.42	-10.71	4.54	-8.28	3.63	-26.67	4.95
30	ϕ	10.1	33.25	6.42	-15.3	6.68	-11.87	6.93	-8.58	5.03	-33.64	7.58
35		22.9	85.73	10.2	-17.27	10.76	-13.22	11.3	-8.35	7.25	-41.2	12.33
40		92.6	327.91	17.5	-19.13	18.86	-12.85	20.1	-7.12	11.03	-47.78	21.64

4. Conclusion

The proposed analysis demonstrates the application of method of slices in which, the only assumption that is required to make the analysis statically determinate is on the variation of direction of inter slice forces. The unique failure surface is fixed using the method suggested by Shields and Tolunay (1973). With the linear variation of the direction of inter slice forces considered in the analysis, computed values of K_P show a reasonably good agreement with some of the available solutions.

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